# A Labor Market Sorting Model of Scarring and Hysteresis*


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**Abstract**

Labor market sorting substantially changes across the business cycle, increasing inefficient mismatches. This paper proposes a new framework to analyze the interactions between labor market sorting and business cycle fluctuations. Using Italian administrative data, we estimate a tractable search equilibrium model that considers firm and worker heterogeneity, aggregate risk, and life-cycle dynamics. We jointly account for endogenous changes in match-dependent human capital accumulation and worker-firm assignment. We show that distortions in human capital accumulation and worker-firm sorting caused by aggregate fluctuations contribute significantly to the persistent effects of business cycles, resulting in economic hysteresis. Aggregate fluctuations have substantial welfare effects, with significant heterogeneity along the earnings, skill, and age distribution. We also find that features of the labor compensation distribution are key predictors of the severity of downturns. Finally, our model is able to account for the increased length of recessions in recent decades.



**Keywords**: Economic hysteresis, Business cycle fluctuations, Human capital accumulation, Labor market sorting, Labor market scarring
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# 1 Introduction

Labor market sorting, that is, the degree to which more-productive workers are assigned to more-productive firms, substantially changes across the business cycle (Lise and Robin, 2017). Recent research has shown that negative impacts on sorting can depress output and labor productivity (Bagger and Lentz 2019; Baley, Figueiredo and Ulbricht 2022). In this paper, we show that analyzing the dynamics of labor market sorting together with business cycle fluctuations and endogenous human capital accumulation can give novel answers to classical questions raised in other work, such as Blanchard and Summers (1986), Cerra, Fatás and Saxena (2023) and Haltiwanger et al. (2022): Why do we observe output hysteresis i.e. the tendency of output dynamics to be more persistent than productivity fluctuations? Do recessions improve or worsen allocative efficiency in the economy? Who bears the costs of business cycles?

To address these questions, we develop a tractable model of the economy in which sorting can endogenously change over the business cycle and estimate it using matched employer-employee data on the universe of the Italian workforce. Our model builds on the directed search models of Menzio and Shi (2010) and Schaal (2017), with an overlapping generations structure (as in Menzio, Telyukova and Visschers, 2016) that permits the characterization of long-term dynamics in workers' careers and labor market participation. We incorporate firm and worker heterogeneity, life-cycle and participation dynamics, search on the job, endogenous human capital accumulation, and aggregate risk. Workers are risk averse and heterogeneous in their human capital level, as well as in their ability to accumulate it on the job. The overlapping-generations structure of the economy allows us to keep track of age and cohort effects separately. These features allow us to understand the role of distributional channels in output fluctuations analysis and at the same time to obtain a realistic characterization of life-cycle earnings profiles, job ladders, and workers' earnings dynamics.

A growing body of literature argues that firm quality is a major determinant of workers' human capital accumulation and skill development (Lise and Postel-Vinay 2020; Arellano-Bover and Saltiel 2022). This means sorting is crucial for understanding human capital accumulation and career paths. In our framework, aggregate fluctuations impact labor market sorting by forcing the dissolution of non-viable worker-firm matches. Because of search frictions, changes to sorting can take a long time to revert. As a result, separations entail considerable losses for workers (Burdett, Carrillo-Tudela and Coles 2020; Huckfeldt 2022; Jarosch 2023) and the model predicts persistent scarring effects on their earnings, human capital, and career prospects (Kahn 2010; Schmieder, von Wachter and Heining 2022; Arellano-Bover 2022).

We present four important results. First, we show that distortions in both human capital accumulation and worker-firm sorting are key factors in explaining the persistent impact of business cycle fluctuations. Second, given the long-lasting effects on workers' careers, we find that fluctuations entail substantial welfare effects on average, with significant heterogeneous effects along the earnings distribution. Third, we show that output dynamics exhibit a considerable degree of state dependence, and higher moments of the labor compensation distribution turn out to be key predictors of the severity of downturns. In particular, we observe that the skewness in the distribution of firms' labor shares is negatively related to output losses from a business cycle shock. Fourth, considering the increase in skill premia and the rising importance of firms for human capital accumulation, our model can account for the lengthening of recessions in recent decades.

In the model, firms produce according to a technology that yields increasing returns in both worker and firm productivity. The value of matches thus depends not only on promised earnings but also on the implied human capital accumulation path during the worker's tenure at the firm. We assume human capital production depends on the match between workers and firms. Workers, both when employed and when unemployed, direct their search toward firms that are heterogeneous in productivity, trading off the value of each vacancy with the likelihood of matching. Human capital is accumulated at a faster pace in jobs at more-productive firms (Lise and Postel-Vinay, 2020), which jobs are harder to obtain. We validate this mechanism by replicating, on both the model and data, the results on the importance of past employers on subsequent earnings obtained by Herkenhoff et al. (2018) and Arellano-Bover (2022).

The model is completed by a fully state-contingent wage protocol that splits the surplus between workers and firms. Profit-maximizing firms design a menu of contracts to attract and retain the most-productive workers whose risk aversion creates a demand for insurance from fluctuations of match value, as in Holmstrom (1983), Balke and Lamadon (2022), and Souchier (2022). The resulting state-contingent contract backloads compensation to maximize retention, while partially insuring workers against idiosyncratic and aggregate shocks. Because workers accumulate human capital on the job, incentives to search on the job vary over job tenure, and so does the wage growth within the match.[1] Given risk aversion, firms' human capital production function, and the incentives embedded in the contract, we prove that the model economy features a monotonically increasing mapping in the search strategies of workers toward firms in equilibrium.

A model that aims to measure the impact of the business cycle on sorting must be

[1] Given the model's treatment of pay dynamics within the worker-firm match, our research is also significantly related to a strand of literature in labor and finance analyzing the firms' management of liquidity and labor compensation dynamics (Xiaolan (2014); Favilukis, Lin and Zhao (2020); Acabbi and Alati (2022); Acabbi, Panetti and Sforza (2023)).

able to accurately characterize endogenous cyclical displacements. To capture the role of cyclical displacements, we incorporate a two-sided commitment friction that generates inefficient separations. A sizeable literature identifies substantial resistance to nominal or real wage cuts (Altonji and Devereux 1999; Agell and Lundborg 2003; Grigsby, Hurst and Yildirmaz 2021). Downward wage rigidity limits the ability of firms to reduce losses when they are hit with aggregate or idiosyncratic shocks, which can lead them to become insolvent and lay off workers in bad times. The model is then able to reproduce empirical separation patterns across age, worker, and firm productivity, and matches the correlation of separations with business cycles. Whenever aggregate conditions change, firms respond endogenously by adjusting their respective wage schedules and job openings while workers optimally adjust their quit and search policies. The resulting pattern of wage growth, matches, and separations is consistent with the data and reproduces a host of results from the empirical labor literature on scarring effects on workers careers (Kahn 2010, Schwandt and von Wachter 2019, Schmieder, von Wachter and Heining 2022, Huckfeldt 2022, Jarosch 2023, Bertheau et al. 2023). The model is thus able to replicate the micro-level evidence and can be used as a laboratory to assess the interactions between business cycles and labor market sorting.

Our first contribution is to analyze the dynamic effects of aggregate fluctuations on output through a decomposition of each channel of labor market adjustment. In the model, the number of displacements constitutes the main driver of output dynamics at the onset of a recession and significantly correlates with the depth of a recession's trough. Slow employment recoveries and sluggish recoveries - output hysteresis - depend instead on a worsened worker-firm sorting, whereby workers match with firms of lower quality which in turn leads to lower human capital accumulation. We apply the logic of this decomposition to the dynamics of the Italian business cycle around the 2011 Sovereign Debt Crisis. Muting the sorting and human capital channel yields a quicker recovery, with GDP back to trend two years earlier than observed in the data and reproduced by the baseline model. The outcome of this exercise shows that recessions have a sullying effect on the economy (Barlevy, 2002): distortions in sorting and human capital accumulation lead to a persistent deterioration in the quality of matches after a recession.

We then show that hysteretical effects determined by distortions in sorting and human capital accumulation in recessions have a sizable impact on welfare. We estimate that the welfare cost of business cycles is, on average, above 2%, broadly in line with estimates in Barlevy (2004) and Dupraz, Nakamura and Steinsson (2022).[2] Our model shows that these costs are highly heterogeneous, with lower-skilled younger workers experiencing business cycles costs that are twice as large as the average. We also show that this result is related to the increase in inequality measured throughout recessions (Heathcote,

[2]Notice that these numbers are two orders of magnitude larger than what was calculated by Lucas (1987).

Perri and Violante, 2020), which our model accurately reproduces. The source of welfare losses differs across the earnings distribution: poorer workers tend to incur welfare losses principally because of displacements. Higher-skilled workers, on the other hand, suffer welfare losses because of the scars on their human capital accumulation path, in prospect a longer term effect.

Our third contribution is to show that cross-sectional movements in labor compensation and match quality following a recession are crucial in determining how susceptible the economy is to subsequent shocks. We find that skewness in the cross-sectional distribution of the labor share can explain the severity of recessions. This finding highlights the importance of labor market fluidity for aggregate dynamics (Engbom, 2020). The wage backloading resulting from our wage protocol implies that salaries never decrease while matches are viable. As workers do not easily switch to higher productivity jobs but accumulate wage increases with tenure in their jobs, a more stagnant labor market reduces the ability of the economy to absorb negative shocks.

Our fourth contribution is to relate the dynamics we detect in sorting and hysteresis to the increase in the length of recoveries from recessions in developed economies in the last thirty years (Fukui, Nakamura and Steinsson, 2023). Recent research has shown that advanced economies have experienced a secular trend toward greater sorting in labor markets (see Song et al., 2019), while skill premia and the importance of firms in human capital accumulation have increased (Arellano-Bover and Saltiel, 2022). We find that most of the increased length observed in advanced economies can be explained by demographic shifts (aging) and a higher dependence on firm quality of worker's human capital accumulation.

Finally, we study the effects of two common labor market policies: the introduction of a minimum wage (Dustmann et al. 2021; Berger, Herkenhoff and Mongey 2022; Vogel 2022) and of a countercyclical transfer to the unemployed, akin to countercyclical unemployment benefits (Nekoei and Weber, 2017). The introduction of a minimum wage has two effects. On the one hand, it negatively affects job creation. On the other hand, low-skilled workers might benefit from its introduction through higher earnings and possibly greater quality jobs. We show that aggregate welfare decreases, as the gains accruing to these workers do not counterbalance the loss in business dynamism. On the other hand, countercyclical unemployment benefits increase long-run output and welfare, mostly by reducing the scarring effects of recessions (Schmieder, von Wachter and Bender 2016; Schmieder, von Wachter and Heining 2022). The expectation of large unemployment transfers improves labor force participation. Transfers also make workers more "patient", and hence more ambitious, in their search strategy. This improves the overall allocative efficiency and increases the stock of human capital.

# 2 Model

## 2.1 Environment

Time is discrete, runs forever and is indexed by $t \in \mathbb{N}$. We denote future values in recursive expressions by adding a $'$ to them, or index elements by $t$ in non-recursive ones.

The economy is populated by $T \geq 2$ overlapping generations of finitely lived, hand-to-mouth, risk-averse workers and a continuum of risk-neutral entrepreneurs. All agents in the economy share the same discount factor $\beta \in (0, 1)$. Each household lives for $T$ periods, with age $\tau \in \mathcal{T} \equiv \{1, 2, 3, \ldots, T\}$. Workers are either employed, with value function $W$, or unemployed, with value function $U$.

Workers maximize lifetime flow-utility from non-durable consumption:

$$\mathbb{E}_{t_0}\left(\sum_{\tau=1}^{T} \beta^{\tau} u(c_{\tau,t_0+\tau})\right),$$

where $t_0$ characterizes the time of entry into the labor market, and $\tau$ characterizes the age of the agent. $c_{\tau,t_0+\tau}$ thus refers to the consumption of workers of age $\tau$ in time $t_0 + \tau$.

Workers are characterized by heterogeneous human capital levels $h$, with $h \in \mathcal{H} \equiv [\underline{h}, \overline{h}]$. Workers are heterogeneous also with respect to their education level $\iota \in \mathcal{I} \equiv \{\mathrm{g}, \mathrm{s}\}$, which indicates college and high school education, respectively. Both types enter the labor market with a baseline level of human capital drawn from type-specific exogenous continuous distributions. Upon entry in the labor market, $\mathbb{E}[h|\mathrm{g}] > \mathbb{E}[h|\mathrm{hs}]$. To account for the different number of education years in the data, graduate workers entry to the labor market is delayed accordingly. Workers exit the labor force when their employment prospects deteriorate below a certain utility threshold. This allows us to distinguish long-term unemployment from non-participation.

Lise and Postel-Vinay (2020) and Xiaolan (2014) show that human capital accumulation can be heterogeneous across firms, and that the maximum level of human capital attainable at each firm is a key variable in determining workers' compensation. We model the human capital accumulation process assuming that workers' gain in human capital depends on the quality of the firm they are matched with and their own initial level of human capital, $h$. Firms are characterized by different levels of quality $y \in \mathcal{Y} \equiv [\underline{y}, \overline{y}]$, which is isomorphic to capital levels. Workers accumulate human capital only while employed and according to a law of motion that is match-specific: $h' = \phi(h, y, \iota, \psi) = g_{\iota}(h, y) + \psi$, $n : \mathcal{H} \times \mathcal{Y} \times \mathcal{I} \to \mathcal{H}$, where $g_{\iota}$ is the deterministic component of the human capital accumulation dynamics, and $\psi$ constitutes the stochastic component. The function $g_{\iota}$ is concave in both its arguments. The deterministic component of human capital accumulation is akin to a "catching-up" of the firm's quality, up to a point when the worker will not be able to learn any more from the match.[3] The only difference between graduates and non-graduates (indexed by $\iota$) is the speed of the "catching-up". Graduate workers are workers that can become more specialized over time, and will thus catch up faster.

Human capital accumulation is risky: at any period any employed worker is subject to the idiosyncratic human capital shock $\psi$, which enters additively with respect to the deterministic component.[4] The shock affects workers' ability and can amplify, shrink, or even reverse human capital accumulation. We further allow for the possibility that human capital deteriorates while workers are unemployed, according to an arbitrary process $g_u$.[5]

Firms are modeled as one worker–one job matches, thus abstracting from firm size. Each job match is characterized by a promised utility to the worker $V \in \mathcal{V}$. We group worker-specific characteristics in a tuple $\chi \in \mathcal{X} \equiv \{\mathcal{H} \times \mathcal{T} \times \mathcal{I}\}$. The aggregate state of the economy $\Omega$ is characterized by the productivity level $a \in \mathcal{A} \subseteq \mathbf{R}_0^+$ and by the distribution of agents across states $\mu \in \mathcal{M} : \{W, U\} \times \mathcal{Y} \times \mathcal{X} \times \mathcal{V} \to [0,1]$. Let $\Omega = (a, \mu) \in \mathcal{A} \times \mathcal{M}$ represent the aggregate state of the economy and let $\mathcal{M}$ represent the set of distributions $\mu$ over the states of the economy. Let $\mu' = \Phi(\Omega, a')$ be the law of motion of the distribution. Aggregate productivity evolves as a stationary monotone increasing Markov process, namely $a' \sim F(a'|a) : \mathcal{A} \to \mathcal{A}$, with the Feller property.

## 2.2 Labor markets

Search is directed. Each labor market is organized as a continuum of submarkets indexed by the expected lifetime utility offered by firms of type $y$, $v_y \in \mathcal{V} \equiv [\underline{v}, \overline{v}]$. Workers are indexed by the tuple $\chi = (h, \tau, \iota)$.[6] The process of starting a firm, which amounts to posting a vacancy at a quality-specific cost $\kappa(y)$, will be described in **Section 2.4**.

The search process is characterized by a constant return to scale, twice continuously differentiable, matching function $M(u, \nu)$ for each submarket. The tightness of each submarket in $\mathcal{X} \times \mathcal{V}$ is defined as $\theta = \nu/u$, with $\theta(\cdot) : \mathcal{X} \times \mathcal{V} \to \mathbf{R}_0^+$. Job finding rates are defined as $p(\theta(\cdot)) = M(u, \nu)/u$, where $p(\cdot) : \mathbf{R}_0^+ \to [0,1]$ is a twice continuously differentiable, strictly increasing, and strictly concave function with $p(0) = 0$, $\lim_{\theta \to +\infty} p(\theta) = 1$ and $p'(0) < \infty$. The vacancy-filling probability is in turn defined as

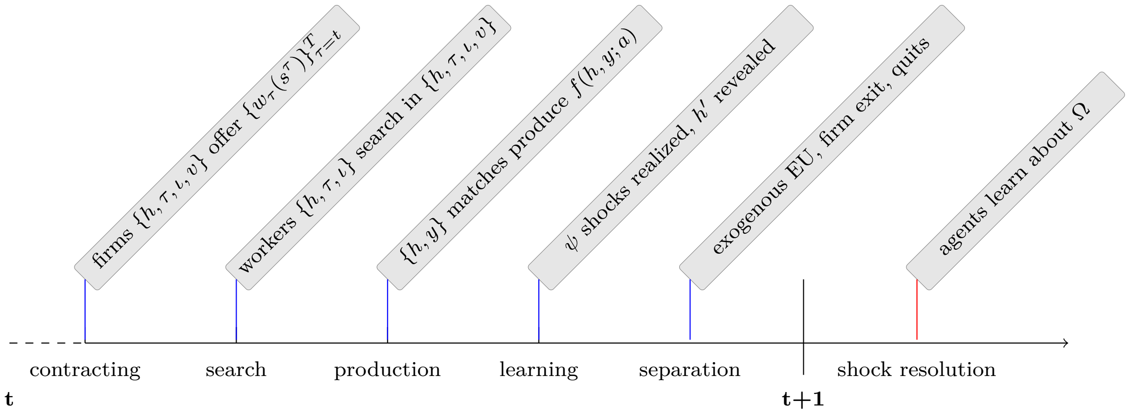
[3] Workers who match with a low-quality firm will see their ability deteriorating with the same $g$ function.

[4] The additive nature of the shock keeps the properties of monotonicity and uniqueness of workers' search strategies unaltered, which is essential for tractability.

[5] This process might be without loss of generality deterministic or stochastic, and might or might not depend on current human capital $h$.

[6] As in Menzio and Shi (2010) the equilibrium will be separating. Given a menu of offers from any firm, each worker will visit only a particular submarket. For this reason submarkets can then be indexed directly by workers' current characteristics (see **Section 2.3**).

**Figure 1.** Timeline of Worker–Firm Match




$q(\theta(\cdot)) = M(u, \nu)/\nu$ , where $q(\cdot) : \mathbf{R}_0^+ \rightarrow [0, 1]$ is twice continuously differentiable, strictly decreasing, and strictly convex, with $q(0) = 1$, $\lim_{\theta \to +\infty} q(\theta) = 0$ and $q'(0) < 0$. Given these properties $q(\theta) = p(\theta)/\theta$, and $p(q^{-1}(\cdot))$ is concave.

Upon match, workers produce according to the twice-continuous increasing and concave production function $f(h, y; a) + x(a) : \mathcal{A} \times \mathcal{H} \times \mathcal{Y} \rightarrow \mathbf{R}_0^+$. The $x(a)$ component of the production function is a fixed cost, which can depend on the aggregate productivity realization.[7] Workers' compensation is determined by means of dynamic contracts through which firms deliver a promised utility, as described in **Section 2.5**.

Workers can search on the job with probability $\lambda_e$. Matches are destroyed at an exogenous rate $\lambda_\tau$ each period, with the exogenous separation rate possibly varying by age. Matches separate also if the worker is poached by another firm, if the worker voluntarily decides to quit and become unemployed (quits), or if the value of the match for the firm becomes negative (firings). Lastly, unemployed workers whose expected value of re-employment falls below a threshold $\underline{\mathrm{p}}$ are assumed to have permanently exited the labor force.

Timing is represented in Figure 1. At the beginning of each period an aggregate productivity shock is drawn; entrepreneurs open vacancies across submarkets and post their offers; workers search from unemployment or on-the-job, and move to a new job if the search is successful; production takes place; workers accumulate human capital depending on their employment status and idiosyncratic shock realization; an exogenous share of matches breaks down, while some firms endogenously exit and some workers quit.

[7]This is a reduced-form way of incorporating financial frictions in the model, which make fixed costs loom larger over flow-production in downturns.

## 2.3 Informational and contractual structure

Firms post fully state-contingent contracts. Each contract prescribes an action for each realization of the history of the worker–firm match. The state of a match at a generic time $t$ is defined by $s_t = (h_t, \tau_t, \iota, a^t, \mu^t) \in \mathcal{S}^t = \mathcal{X} \times \mathcal{Y} \times \Omega^t = \mathcal{H} \times \mathcal{T} \times I \times \mathcal{Y} \times \Omega^t$, that is the worker skill, age, education, the history of aggregate productivity shocks, and workers' distributions across their employment history. A given history of realizations between $t$ and $k$ periods ahead is thus $s^{t+k} = (s_t, s_{t+1} \ldots, s_{t+k})$. The contract defines a transfer of utility from the risk-neutral firm to the risk-averse worker within the match for all future possible histories of shocks. We define $\tau_{t_0}$ as the age at which the worker is hired and $T$ is the retirement age. The history of realizations between $t_0$, the time of hiring of the worker, and $t_0 + (T - \tau_{t_0})$, the time of maximum duration of the match with the worker before retirement, is thus $s^{t_0+(T-\tau_{t_0})}$.

Histories of workers and productivity shocks are common knowledge, and the future realizations of shocks are fully contractible. While the contract is state-contingent, markets are incomplete: workers' actions are private knowledge in the search stage, so firms are unable to directly counter outside offers. The contracts offered by firms are then defined as:

$$\mathcal{C}^{\tau_{t_0}} := (\mathbf{w}, \zeta) \text{ with } \mathbf{w} := \{w_t(s^{\tau_t - \tau_{t_0} + t_0})\}_{t=t_0}^{t_0+(T-\tau_{t_0})}, \text{ and } \zeta := \{v_t(s^{\tau_t - \tau_{t_0} + t_0})\}_{t=t_0}^{t_0+(T-\tau_{t_0})}. \tag{1}$$

Firms promise a series of state-contingent wages defined by the series of utility values $v_t$ sought at each node of the history.[8] $\zeta$ is the action suggested by the contract, which is bound to be incentive compatible for the worker. The resulting relationship between workers and firms is characterized by a contract with forward-looking constraints. The state space of the worker problem can be expressed in terms of their current lifetime utility, as in Spear and Srivastava (1987), so as to avoid having to keep track of all past histories $s^t$ at each period. The relevant state space is then $\mathcal{X} \times \mathcal{V}$.

## 2.4 Vacancy creation and free entry

The economy is populated by a continuum of risk-neutral entrepreneurs. Each entrepreneur can invest to reach the desired level of firm quality $y$. The start-up costs of the firm are priced in terms of the consumption good and they coincide with vacancy posting costs in the frictional labor market. The cost of each vacancy is positively related to the quality of the firm being created. To post a vacancy for the creation of a firm with quality $y$ the entrepreneur must thus pay $c(y)$, a vacancy cost priced in terms

[8] Similarly to Menzio and Shi (2010), Tsuyuhara (2016), and Balke and Lamadon (2022), to guarantee that the problem is well behaved and the firm profit function is concave, the contract will require a two-point lottery, which specifies probabilities over the actions prescribed. We omit it here for conciseness.

of the consumption good. The vacancy cost function $c(y)$ is a strictly convex function of firm quality $y$.[9]

At a generic time $t$ each entrepreneur chooses in which submarket to post the vacancy selecting a lottery over the offered utility $W$, which maps into the set of firms' qualities $y \in \mathcal{Y}$, and worker characteristics $(\chi, V) \in \mathcal{X} \times \mathcal{V}$.

We define $J(h, \tau, \iota, W, y; \Omega) \in \mathcal{X} \times \mathcal{V} \times \mathcal{Y} \times \Omega$ as the value function of a firm, which capitalizes all future profits from the match. As entrepreneurs choose the submarkets in which to open a vacancy, they face the following problem:

$$\Pi(h, \tau, \iota, W, y; \Omega) = \sup_{y,h,\tau,\iota,W} \; - c(y) + q(\theta(h, \tau, \iota, W; \Omega))[J(h, \tau, \iota, W, y; \Omega)] \tag{2}$$

Given perfect competition, free entry and the possibility for all entrepreneurs to choose *any* possible firm kind $y$, the expected profits from creating a vacancy are driven down to 0 in submarkets that actually open.[10] This translates into a free entry condition:

$$\Pi(h, \tau, \iota, W, y; \Omega) \leq 0 \;\text{ for } \forall\{h, \tau, \iota, W, y; \Omega\} \in \{\mathcal{X} \times \mathcal{V} \times \mathcal{Y} \times \Omega\} \tag{3}$$

Assuming that $q(\cdot)$ is invertible, the equilibrium tightness in each submarket is:

$$\theta(h, \tau, \iota, W; \Omega) = q^{-1}\left(\frac{c(y)}{J(h, \tau, \iota, W, y; \Omega)}\right). \tag{4}$$

## 2.5 Firm problem

Firms commit to the delivery of a utility value to workers, but exit when the present value of future profits becomes negative. Workers' limited commitment implies they will search for new jobs whenever they have the possibility to do so. Firms cannot observe poaching offers and thus cannot counteract them. The sequence of past histories $s^t$ is common knowledge, and while the firm cannot observe any of actions of its workers, it has enough information to internalize their optimal search policy decisions.

Define $\widetilde{p}(\chi, V; \Omega)$ as the optimal retention function and $\widetilde{r}(\chi, V; \Omega)$ as the optimal utility return from workers' solution to the on-the-job search problem. The value function of an incumbent firm $y$ in state $(h, \tau, \iota, W_y; \Omega)$ can be written recursively using

[9] We assume that entrepreneurs can borrow from risk-neutral, deep-pocketed financiers to finance the vacancy. In Herkenhoff (2019) this assumption implies the cost of credit for entrepreneurs to coincide with the risk-free rate.

[10] Notice that in this case the expectation does not refer to realizations of the aggregate state $\Omega$ or the human capital shock $\psi$, but to the vacancy-filling probability $q$.

the promised utilities as additional state variables as:

$$
\begin{aligned}
&J(h,\tau,\iota,W,y;\Omega)= \\
&\quad \sup_{\pi_i,\{w_i,W_i'\}} \sum_{i=1,2}\pi_i\Bigg(f(y,h;a)-w_i \\
&\quad +\beta\mathbb{E}_\psi\left[\max\left\{0,\mathbb{E}_\Omega\left[\widetilde{p}(h',\tau+1,\iota,W_i';\Omega')J(h',\tau+1,\iota,W_i',y;\Omega')\right]\right\}\right]\Bigg)
\end{aligned}
\tag{5}
$$

$$
s.t.\ W=\sum_{i=1,2}\pi_i\left(u(w_i)+\beta\mathbb{E}_{\Omega,\psi}\left(\widetilde{r}(h',\tau+1,\iota,W_i';\Omega')\right)\right), \tag{6}
$$

$$
\sum_{i=1,2}\pi_i=1, \tag{7}
$$

where **Equation** (6) is the promise keeping constraint ensuring that the current value of the contract is based on the current wage and future utility promises with $\widetilde{r}_t(\cdot)$. The firm (the principal) chooses the wage(s) to be offered in the current period $w_i$, the utility promises $W_i'$ and the probability $\pi_i$ in the two-point lottery. The optimization implicitly takes into account the utility of workers (the agent) and their incentive compatible best replies, through the retention probability $\widetilde{p}(\cdot)$ and the expected utility gain $\widetilde{r}(\cdot)$. The continuation value for an incumbent firms allows for the possibility of dissolving the match if its value falls below zero.

Incumbent firms make their exit decisions before the realization of aggregate productivity but after the realization idiosyncratic human capital shocks for the next period.[11] At the beginning of a period they already know whether they will exit. Exit is therefore completely determined by the current state and can be summarized by a threshold for the aggregate productivity level and can be described as follows.

**Definition 2.1** (Exit policy)**.** *The following indicator takes a value of one if the firm does not decide to exit in the following period:*

$$
\eta(h,\tau,\iota,W,y;\Omega,\psi)=\begin{cases}1 & \text{if } a\geq\max\{0,a^*\}\\ 0 & \text{otherwise}\end{cases}
$$

*with the productivity threshold defined as*

$$
a^*(h,\tau,\iota,W,y;\Omega,\psi):\mathbb{E}_\Omega[J(h',\tau+1,\iota,W',y;\Omega')]=0. \tag{8}
$$

[11] This amounts to having the firm making a state-contingent exit decision in advance of the idiosyncratic shock's realization as in Gomes (2001) and Xiaolan (2014).

## 2.6 Worker problem

Given current lifetime utility $V$, job seekers with characteristics $\chi$ have to decide in which submarket to direct their search. Submarkets are indexed by worker type $\chi$ and by offered utility $v$ associated to firms' posted vacancies. As discussed in **Section 2.4**, the choice over $v$ will also indirectly determine which kind of firm $y$ the worker matches with, and thus the implied human capital accumulation path. For now, let us assume this (conditional) mapping exists. This amounts to assuming that the function $v(y;\chi,V)$ is an injective function $f_v : \mathcal{Y} \times \mathcal{X} \times \mathcal{V} \to \mathcal{V}$. Upon observing a job offer with utility $v$, a worker $\chi$ with current utility $V$ will be able to infer which firm type $y$ is posting the offer.

A worker of type $(\chi, V)$ that enters the search stage has lifetime utility $V + \max\{0, R(\chi, V;\Omega\}$, where the second component of the expression embeds the option value of the search, with $R$ being the search value function. $R$ is defined as:

$$R(\chi, V;\Omega) = \sup_{v} \Big[ p(\theta(\chi, v;\Omega)) \big[ v - V \big] \Big]. \tag{9}$$

We denote the solution of the search problem as $v^* = v^*(\chi, V;\Omega)$, and $p^*(\chi, v^*;\Omega) = p(\theta(\chi, v^*;\Omega))$ as the associated optimal job-finding probability. The lifetime utility of an unemployed worker at the beginning of the production stage can be defined as

$$\begin{aligned} U(h,\tau,\iota;\Omega) = \; & u(b(h,\tau)) + \beta \mathbb{E}_{\Omega,\psi} \Big( U(h',\tau+1,\iota;\Omega') \\ & + \max\{0, R(h',\tau+1,\iota, U(h',\tau+1,\iota;\Omega');\Omega')\} \Big), \end{aligned} \tag{10}$$

where $b(h,\tau)$ is a skill and age dependent unemployment benefit. Given finite workers' lives, $U(h,\tau,\iota;\Omega) = 0 \; \forall (\chi;\Omega) \in \mathcal{X} \times \Omega$ whenever $\tau > T$. The corresponding lifetime utility of a worker employed at firm $y$, with human capital $h$, age $\tau$, education $\iota$ and promised utility $V$ at the beginning of production stage can be expressed as:

$$\begin{aligned} V(h,\tau,\iota;\Omega) = \; & u(w) + \beta \mathbb{E}_{\Omega,\psi} \Big( \lambda_\tau U(h',\tau+1,\iota;\Omega') + \; (1-\lambda_\tau) \Big[ V(h',\tau+1,\iota;\Omega') \\ & + \; \lambda_e \max\{0, R(h',\tau+1,\iota, V(h',\tau+1,\iota;\Omega');\Omega')\} \Big] \Big), \end{aligned} \tag{11}$$

where $w$ is the promised wage and $V(h',\tau+1,\iota;\Omega')$ is next period's state-contingent promised utility of remaining in the current firm, which becomes the outside option in the search problem.[12] Firms internalize incentives embedded in workers' strategies and

[12]It is here implied that, in case there is an endogenous separation, this future promised value is

post wages and utility offers to maximize profits by optimizing retention. This way, future promised utilities incorporate both future wages or option values of search.

The policy functions are uniquely defined and allow us to identify target $y$ as long as the injective mapping between the offered utility $v$ and $y$ given $\chi$ exists.[13]. The solution of employed workers' on-the-job search problem defines a search policy function. In turn, this policy function leads to the definition of two equilibrium objects, which firms internalize in their optimization in order to incorporate workers' incentive compatibility.

**Definition 2.2** (Optimal retention probability and utility return)**.** *The solution to the worker's problem defines a retention function* $\widetilde{p} : \mathcal{X} \times \mathcal{V} \times \Omega \to [(1-\lambda)(1-\lambda_e), 1-\lambda]$ *and a utility return* $\widetilde{r} : \mathcal{X} \times \mathcal{V} \times \Omega \to \mathcal{V}$*:*

$$\widetilde{p}(\chi, V; \Omega) \equiv (1-\lambda_\tau)(1-\lambda_e p^*(\chi, v^*; \Omega)) \tag{12}$$

$$\widetilde{r}(\chi, V; \Omega) \equiv \lambda_\tau U(\chi; \Omega) + (1-\lambda_\tau)\Big[V + \lambda_e \max\{0, R(\chi, V; \Omega)\}\Big] \tag{13}$$

## 2.7 Equilibrium definition

**Recursive Equilibrium.** Let $\Theta = \mathcal{A} \times \mathcal{M} \times \mathcal{H} \times \mathcal{T} \times \mathcal{I}$. A recursive equilibrium in this economy consists of a market tightness $\theta : \Theta \times \mathcal{V} \to \mathbb{R}_+$, a search value function $R : \Theta \times \mathcal{V} \to \mathbb{R}$, a search policy function $v^* : \Theta \times \mathcal{V} \to \mathcal{V}$, an unemployment value function $U : \Theta \to \mathbb{R}$, a firm value function, $J : \Theta \times \mathcal{V} \times \mathcal{Y} \to \mathbb{R}$, a series of contract policy functions $\{c_\tau\}_{\tau=1}^{T} : \mathcal{S}^\tau \times \mathcal{Y} \to \mathcal{C}^\tau$, an injective mapping between firm qualities and promised utilities at hiring $f_v : \mathcal{X} \times \mathcal{V} \times \mathcal{Y} \to \mathcal{V}$, an exit threshold for aggregate productivity $a^* : \mathcal{X} \times \mathcal{V} \times \mathcal{Y} \to \mathcal{A}$, a human capital accumulation process $\phi(h, y, \iota, \psi)$, $\mathcal{H} \times \mathcal{Y} \times \mathcal{I} \times \Psi \to \mathcal{H}$, and a law of motion for the aggregate state of the economy $\Phi_{\Omega,a} : \mathcal{A} \times \mathcal{M} \to \mathcal{A} \times \mathcal{M}$ such that:

1. Given the mapping $f_v$, market tightness satisfies **Equation** (4).
2. The unemployment value function solves **Equation** (10).
3. Search value functions solve the search problem in **Equation** (9) and $v^*$ is the associated policy function.
4. Firm value functions and associated contract policy functions solve **Equation** (5) for each $t \leq T$.
5. The exit threshold satisfies **Equation** (8).

equivalent to the value of being unemployed.

[13]Proofs of the uniqueness of policy functions and individuals' optimal policy are provided in Online Appendix Section 3

6. The law of motion for the aggregate state of the economy respects the search and contract policy functions and the exogenous process of aggregate productivity.

**Definition 2.3** (Block Recursive Equilibrium)**.** *A Block Recursive Equilibrium (BRE) is a recursive equilibrium such that the value and policy functions depend on the aggregate state only through aggregate productivity, $a \in \mathcal{A}$ and not through the distribution of agents across states $\mu \in \mathcal{M}$.*

# 3 Discussion

The objective of our model of dynamic sorting is to understand the properties of firm creation and worker search in a setting with two-sided heterogeneity. The following properties guarantee a high degree of tractability.[14]

**Property 3.1** (Unique Injective Mapping)**.** *Upon matching, firm quality $y$ and utility promises in vacancy postings $v$ are related by an injective mapping conditional on the aggregate state of the economy, $\Omega$, and workers characteristics $(\chi, V)$.*

The previous proposition establishes that workers' directed search toward promised values is equivalent to directed search toward firms' types. We can focus on the properties of the search strategy to get a complete view of how sorting works in equilibrium.

**Property 3.2** (Search Monotonicity and Uniqueness)**.** *The optimal search strategy when unemployed, conditional on age $\tau$ and the aggregate state $\Omega$, is unique and weakly increasing in workers' characteristics $(h, \iota)$. The optimal search strategy when employed, conditional on age $\tau$ and the aggregate state $\Omega$, is unique and weakly increasing in workers' characteristics $(h, \iota)$ and current level of lifetime utility $V$.*

**Property 3.2** guarantees that, abstracting from idiosyncratic as well as aggregate shocks, workers sort positively with respect to their education and human capital. **Property 3.1**, in turn, guarantees that workers agree on firms' relative ranking. Firms are thus vertically differentiated, and there is a separating equilibrium whereby workers with different characteristics optimally search in distinct firms.

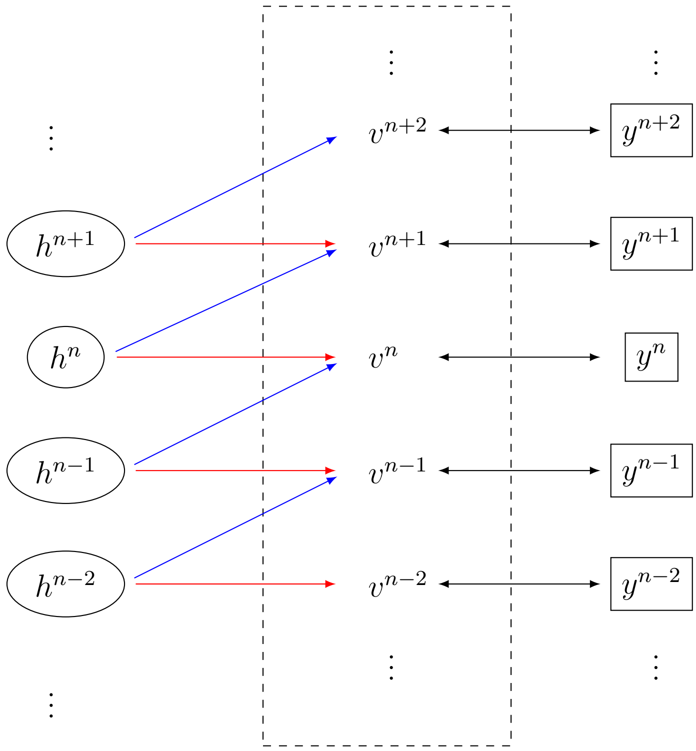
Because we are interested in how aggregate fluctuations shape the distribution of matches, we now turn to changes in search strategies across aggregate states.

**Property 3.3** (Search in Good and Bad Times)**.** *The optimal search strategy is increasing in the aggregate productivity level, $a$.*

[14] We report the essential proofs to the propositions discussed in this section in **Appendix B** and we refer the reader to the Online Appendix for a more in-depth discussion of the theoretical properties of the model.

At this point we are able to illustrate one of the main mechanisms of the model, which is represented in **Figure 2**. The figure highlights one way in which aggregate fluctuations modify sorting in the labor market. The value of vacancies posted by each firm in equilibrium changes with the business cycle, as submarkets become less tight in bad times. Faced with a lower probability of successfully matching with the firm they would aim to match with in good times, risk-averse workers will then adjust their search downward.

**Figure 2.** Search Behavior.




**Note**: Schematic representation of labor market sorting along the business cycle. Unemployed workers, ordered by human capital levels, search in bad times and good times toward values offered by the (unique) corresponding firm type, presented as an ordered list with respect to order n.

Firms' offers will optimally respond to workers' incentives for on-the-job search.

**Property 3.4** (Optimal Retention)**.** *Retention probabilities, $\widetilde{p}(h, \tau, \iota, W; \Omega)$ are:*

*(i) increasing in the value of promised utilities, $W$*

*(ii) decreasing in aggregate productivity, $a$*

Despite continuation values within the match being procyclical and workers searching more ambitiously in good times, firms are more likely to see workers leave in times of expansion. This is consistent with the data, as employment-to-employment transitions are strongly pro-cyclical. **Property 3.4** highlights another important aspect of the incentives that shape the contract designed by firms: retention grows in continuation values $W$.

To close the model, we need a rule for surplus sharing between firms and workers, that is, a wage protocol for firms to deliver lifetime utility promises to workers.

**Property 3.5** (Wage Protocol)**.** *The optimal contract delivers a wage that satisfies:*

$$\frac{\partial \log \widetilde{p}(\chi', W_i'; \Omega')}{\partial W_i'} J(\chi', W_i', y; \Omega') = \frac{1}{u'(w_i')} - \frac{1}{u'(w_i)}, \tag{14}$$

*with* $\chi' \equiv (\phi(h, y, \iota, \psi), \tau + 1, \iota)$ *being the definition of individual characteristics and* $w_i'$ *being the wage paid in the future state, conditional on realizations of idiosyncratic risk* $\psi$ *and aggregate risk* $a'$*.*

This result extends the wage equation in Balke and Lamadon (2022) to an environment with two-sided heterogeneity. Wage growth is proportional to the residual continuation value of the match, $J$ and the semi-elasticity of the worker's retention probability to future value promised. Limited liability provides the rationale for inefficient separations. At the same time, it also gives rise to wage rigidity, as it ensures that both elements in Equation 14 are weakly positive if the firm does not close down.[15]

**Property 3.6** (Countercyclical Separations)**.** *Conditional on the existing contract and on worker and firm types, there exists an aggregate state* $a^*$ *below which firms will not continue the contract. The threshold* $a^*$ *is, all things being equal, increasing in the value promised to workers, and decreasing in worker and firm types.*

A clear implication of **Property 3.6** is that, at the onset of recessions, firms are significantly more likely to lay off workers. In addition, lower-skilled workers and low-productivity firms are more likely to separate in recessions. The counter-cyclicality of separations is a common feature in labor market data, together with the lower job security enjoyed by workers who are less productive, or provided by firms that are less productive .

# 4 Bringing the Model to the Data

The model features internally and externally calibrated parameters. To estimate the first group of parameters, we target moments from Italian administrative data, provided by the Uniemens dataset of the Italian Social Security Administration (INPS), for all years between 1996 and 2018.[16] To obtain model moments, we simulate a population of

[15]Notice that, in the presence of risky human capital accumulation, $J$ will fluctuate together with the human capital levels of the worker even in the absence of aggregate fluctuations. However, because the contract provides insurance to workers, changes in their human capital will have asymmetric effects on wage growth, thus weakly increasing the labor share over time.

[16]Details of data construction and sources are discussed in Online Appendix Section 6.

overlapping generations working for 45 years (180 quarters, from 18 to 63 years old, the legal retirement age for most years in our period of analysis). We then use a simulated method of moments (SMM) approach. This section will first present the quantitative setup of the model, present calibration choices for the parameters that are set externally, and finally present our estimation results.

### 4.1 Calibration and estimation

**Quantitative Setup.** **Table 1** collects all the functional form choices. We assume a Cobb-Douglass production function and allow for potentially cyclical maintenance costs, captured by the parameter $x$. We follow Schaal (2017) and Menzio and Shi (2010) in picking a CES function in market tightness. Vacancy creation imposes increasing costs in firm's quality $y$, according to the convex function $c(y)$. Workers are risk-averse with constant-relative-risk-aversion (CRRA) utility. The human capital production technology is concave in the firm quality, $y$, which is scaled by a parameter $\xi$, and in the existing stock of human capital, $h$. Future human capital is also subject to additive i.i.d. shocks, $\psi \sim \mathcal{N}(0, \sigma_\psi)$. Home production is increasing in the stock of human capital according to the parameter $\xi_b$. Finally, we allow the exogenous separation rate to be age dependent to capture age-specific aspects of worker quality that are unrelated to business cycles but still empirically relevant. The model is then characterized by seven externally calibrated parameters and by 18 jointly estimated parameters.[17]

**Table 1.** Functional Forms

| **Functions** | |
|---|---|
| Production function | $f(y,h) = Ay^\alpha h^{1-\alpha} - x(A-1)$ |
| Job-finding probability | $p(\theta) = \theta(1+\theta^\gamma)^{-\frac{1}{\gamma}}$ |
| Vacancy creation cost | $c(y) = \frac{y^\kappa}{\kappa}$ |
| Utility function | $U(c) = \frac{c^{1-\nu}}{1-\nu}$ |
| Human capital accumulation | $g_\iota(h,y) = (\xi y)^{\phi_\iota} h^{1-\phi_\iota} + \psi$ |
| Home production | $b(h,\tau) = b + \xi_b h$ |
| Exogenous exit rate | $\lambda = \frac{\lambda_b}{\lfloor \tau/4 \rfloor}$ |

**Calibration.** Preference parameters (discount factor $\beta$, and agents' risk aversion $\nu$), and the annualized risk-free rate $r_f$ are set in line with the literature. We calibrate the persistence and volatility of the aggregate shock, ($\rho_a$, $\sigma_a$) by estimating an AR(1) on the detrended series of Italian real total factor productivity (TFP). In addition, workers draw their innate ability and human capital upon entry into the market from an initial distribution. We set this initial distribution of human capital for high school–educated workers as a $Beta(\mu_L, \phi_L)$. College-educated workers draw their initial human capital

[17]Online Appendix Section 7 provides more details on model solution and estimation procedure.

from the same distribution plus a constant spread, $\vartheta$. We set the shape and scale of the beta distribution and internally estimate the scaling factor to match the ratio of average initial incomes between the two groups of workers. Finally, to properly account for the empirical age distribution, we weigh simulated data according to the age distribution of the Italian working-age population.[18]

**Table 2.** Parameter Values

| Parameter | Description | Value |
|---|---|---|
| | **Externally Calibrated** | |
| $\nu$ | Risk aversion | 2.000 |
| $\beta$ | Discounting | 0.990 |
| $r_f$ | Real interest rate | 0.011 |
| $(\mu_L, \sigma_L)$ | Shape and scale of initial human capital dist. | (2.50,10.00) |
| $(\rho_A, \sigma_A)$ | Mean and st.d. of TFP process | (0.95, 0.009) |
| | **Jointly Estimated** | |
| $\alpha$ | Production function elasticity to firm quality | 0.552 |
| $\gamma$ | Matching function | 1.090 |
| $\phi$ | Human capital adjustment rate, High School | 0.037 |
| $\phi_g$ | Human capital adjustment rate, College | 0.282 |
| $b$ | Unemployment benefit | 1.103 |
| $\lambda_b$ | Exogenous separation prob., initial | 0.116 |
| $\kappa$ | Vacancy cost | 2.432 |
| $\lambda_e$ | On-the-job-search prob. | 0.454 |
| $\xi$ | Scaling factor in human capital accumulation | 0.640 |
| $\xi_b$ | Unemployment consumption dependence on human capital | 0.063 |
| $l$ | Linear loss of humanc capital while unemployed | 0.167 |
| $\tau_{ee}$ | Human capital retention after job-to-job transition | 0.902 |
| $\tau_{eu}$ | Human capital loss after displacement | 0.771 |
| $x$ | Cyclical component of cost function | -1.647 |
| $\underline{p}$ | Out-of-labor-force threshold | 0.037 |
| $\sigma_\psi$ | St.d. of idiosyncratic human capital shock | 0.684 |
| $\underline{y}$ | Lowest bound of firm distribution | 2.844 |
| $\vartheta$ | Rescaling of initial human capital, College | 0.350 |

**Estimation.** We estimate the remaining 18 parameters via SMM, targeting a set of standard labor market moments: labor market flows by age, as well as their correlations with aggregate output; the profile of wage growth over workers' careers; the average unemployment rate; the average inactivity rate in the Italian labor market; the average degree of sorting between workers and firms; and the distribution of firms' value-added. We define sorting as the average over time of the correlation between the firm and worker fixed effects from an AKM model yearly estimated on the Italian administrative data. In the model, sorting is the correlation between firms' and workers' qualities. **Table 2** reports the estimated parameter values.

**Fit.** **Table 3** summarizes the comparison of model and empirical moments. The model fits employment flows by age, capturing labor market dynamism in the data (see Engbom, 2020), and we are also able to match the cyclical properties of these flows to account for the jump in job destruction and the drop in job creation during recessions. Characteristics of labor market fluidity and dynamism are further disciplined by matching both the

[18] Age weights are constructed following the age distribution of the 2010 census from the website of the Italian National Institute of Statistics (ISTAT).

unemployment and the inactivity rate. The model also reproduces the life-cycle wage growth of Italian workers by education levels.

**Table 3.** Target Moments

| Moments | Mean | |
|---|---|---|
| | Data | Model |
| A. Labor Market Flows | | |
| Employment-to-Employment Transition Rate* | 1.3% | 0.7% |
| Employment-to-Unemployment Transition Rate* | 4.1% | 2.9% |
| Employment-to-Unemployment Correlation w/GDP | 0.68 | 0.38 |
| Employment-to-Unemployment Correlation w/GDP | -0.19 | -0.16 |
| B. Earnings | | |
| Earnings Growth (High School)* | 129.7% | 134.7% |
| Earnings Growth (College)* | 60.7% | 77.1% |
| Entry Salary Ratio: College to High School | 1.46 | 1.00 |
| C. Other Statistics | | |
| Unemployment Rate | 9.7% | 9.1% |
| Labor Market Sorting | 0.40 | 0.55 |
| Inactivity Rate† | 21.9% | 28.3% |
| Firm Value Added Distribution** | 1.18 | 1.38 |

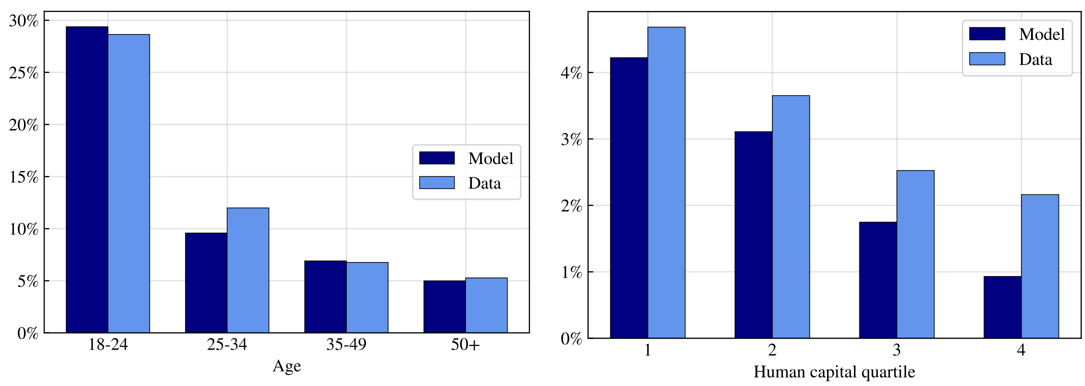
**Note**: (*): We match the life-cycle profiles, with seven age bins for each profile. The table reports average values. (**): We match the ratio between the third and fourth to the first quintiles. (†) The average inactivity rate is from the inactivity rates by age groups from 1996 to 2019 from ISTAT.

## 4.2 Untargeted moments and model features

**Cross-sectional properties.** The model exhibits a very good fit for the age profile of the unemployment rate (untargeted, **Figure 3a**). **Figure 3b** compares the separation

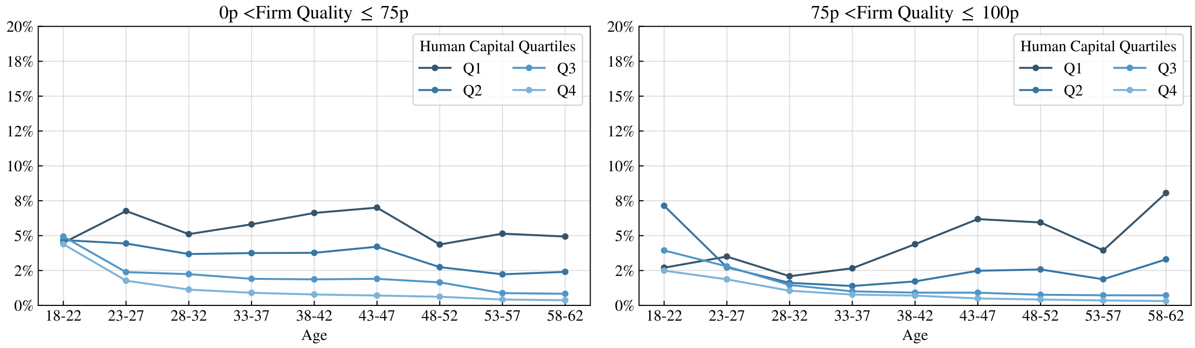
**Figure 3.** Cross-Sectional Features, Model and Data

**(a)** Unemployment rates **(b)** Separation rates by worker qualities




**Note:** Panel (**a**) plots the unemployment rate by age groups in model simulations and in the data. *Sources:* unemployment rates are taken from the Italian National Statistical Agency (ISTAT). Panel (**b**) reports the average separation rates by worker quality. In the data worker quality is measured by the worker-specific AKM fixed effect. In the model simulations, worker quality is workers' human capital.

**Figure 4.** Separation Rates by Age, Firm and Worker Qualities




**Note:** The figure plots the average separation rates by firm quality, human capital quartiles and age in model simulations.

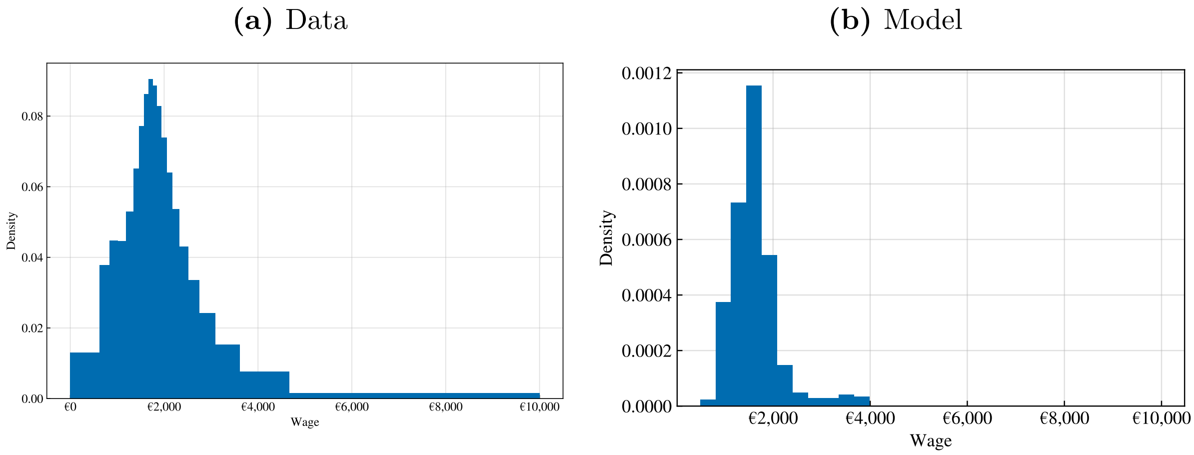
rates by worker types in the model and in the data, highlighting how the model is able to well capture the higher fragility of workers with low human capital, despite not directly targeting these moments in the estimation. **Figure 4** reports the separation rates by firm quality, human capital quartiles, and age bins. Each panel reports the proportion of matches that are dissolved, on average, by age and human capital in each firm class.

The patterns that emerge inform the operating leverage dynamics at work (Favilukis, Lin and Zhao 2020; Acabbi, Panetti and Sforza 2023). Separations are prevalent for low-skilled workers, especially if they are strongly mismatched in relatively good firms. Older workers are more likely to command higher wages thanks to their longer labor market experience, which makes firms more susceptible to aggregate shocks. This mechanism is stronger for more-productive firms, as they also provide a steeper accumulation of human capital, and consequently a steeper wage profile. Separation rates are higher for younger workers, as they are more likely to be in the lowest human capital quartile. In our model, the relatively larger fragility of young workers is mostly due to their small initial endowment of human capital. Changes in the cross-sectional distribution of human capital and labor compensation could thus thus strongly impact the susceptibility of the economy to fluctuations.

The model is also able to qualitatively reproduce the distribution of earnings. **Figure 5** displays the cross-sectional distribution of earnings in the data and in the model. The empirical wage distribution is centered at slightly below €$2,000$ and skewed to the left, with most observations below €$4,000$. What the model fails to generate is the long right tail of wages in the data, which corresponds mainly to managerial figures whose earnings command premia that our mechanism is not meant to capture.

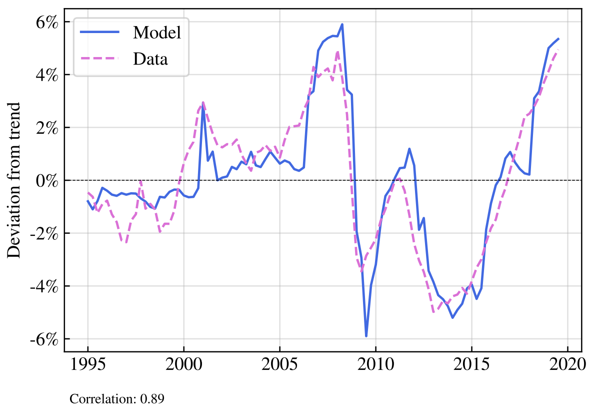
**Time Series Properties.** Replicating aggregate time-series properties of the data provides an important validation of the channels in the model. We use the detrended quarterly series of Italian TFP, and project it on a discrete grid to simulate a series of

**Figure 5.** Wage Distributions




**Note:** The figure plots the wage distributions in the data and in model simulations.

**Figure 6.** GDP: Model and Data



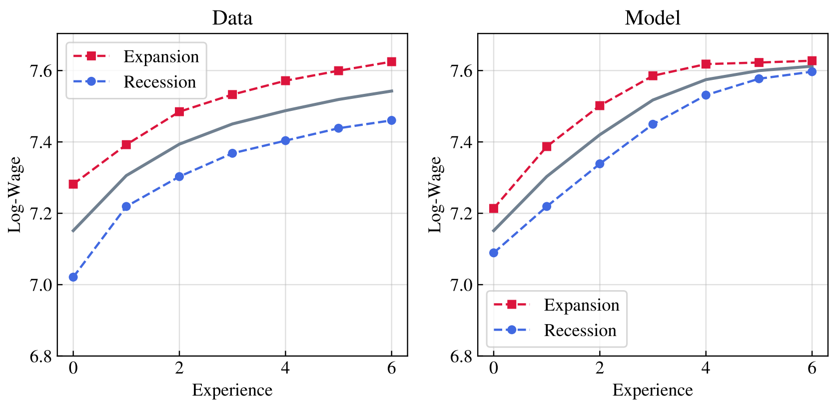

**Note:** The figure plots the cyclical components of real GDP for Italy and for a model simulation in which the TFP process is matched to the Italian TFP realizations from 1995 to 2019, both series are quadratically detrended and their correlation is robust to the choice of the filter.

aggregate shocks in the model. **Figure 6** plots the resulting simulated aggregate production, together with the detrended GDP process from the data. The model replicates the amplification channels from productivity shocks to GDP.

**Scarring.** The model is also able to replicate the long-run effects of business cycles on workers' career outcomes at the micro level. In particular, we adapt the reduced-form models proposed in the literature about the scarring effects of recessions on labor market entrants (Kahn 2010, Schwandt and von Wachter 2019) and we run it on both the Italian administrative data and on a model-simulated panel.[19] Consistently with the literature, entering the labor market in a downturn is associated with persistent losses in earnings.

[19]In these empirical specifications we control for age, period, and cohort effects. Following the literature, we address the well-known identification issues in this class of models by proxying cohort fixed effects with the cyclical realization of GDP. We report the empirical estimates in the Online Appendix.

**Figure 7.** Scarring Effect of Recessions




**Note**: The figure plots the wage profiles estimated on the data and on model simulations for cohorts of workers entering the labor market. The counterfactual profiles for expansions (recessions) are obtained considering a positive (negative) two standard deviation realization of cyclical GDP.

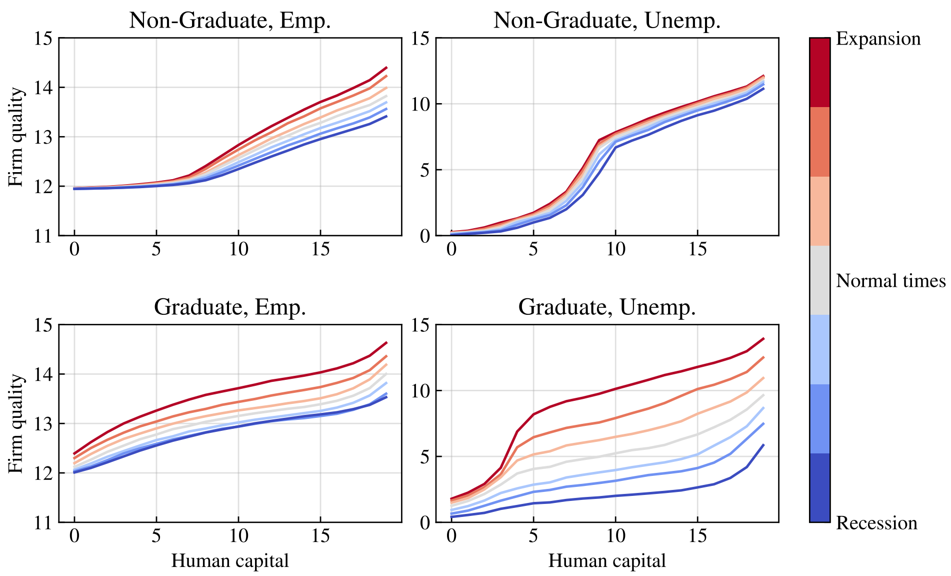
As shown in **Figure 7**, our model is able to replicate well the dynamics of these scarring effects of business cycles.

**Sorting.** The model predicts that workers' search is monotonic in individual characteristics and in the aggregate state (see **Proposition 3.2**). In **Figure 8** we plot the equilibrium mapping between workers' human capital and search behavior at each point in the aggregate state. Search is strongly monotonic in both dimensions. Search strategies of college-educated workers are more sensitive to shifts in aggregate conditions. Because expected human capital accumulation in their matches is higher, the duration of corresponding firm profits is longer. This makes vacancy creation more volatile with respect to shifts in aggregate conditions. Productive jobs that would enhance human capital accumulation become scarcer, and workers respond by moderating their job search strategy, thus increasing misallocation in downturns. Exploiting the rich features of the model, in **Figure 9**, we decompose the average growth in wages *within* jobs, that is, within the same job spell, and *between* jobs, that is, after a job-to-job transition.

Consistent with the data, the model simulation implies that the bulk of wage growth is due to job-to-job transitions. We also observe that average *within*-wage growth is declining in age and firm quality. This is due to the fact that search frictions make it difficult for workers to reach progressively higher-productivity firms. Those firms thus enjoy relatively greater retention probabilities without the need of substantially adjusting wages upwards (as in Gouin-Bonenfant, 2022). The resulting wage profiles for higher-productivity firms are thus flatter. Given that the highest skill workers tend to be older, one obtains decreasing wage growth over the life cycle and with firm quality.

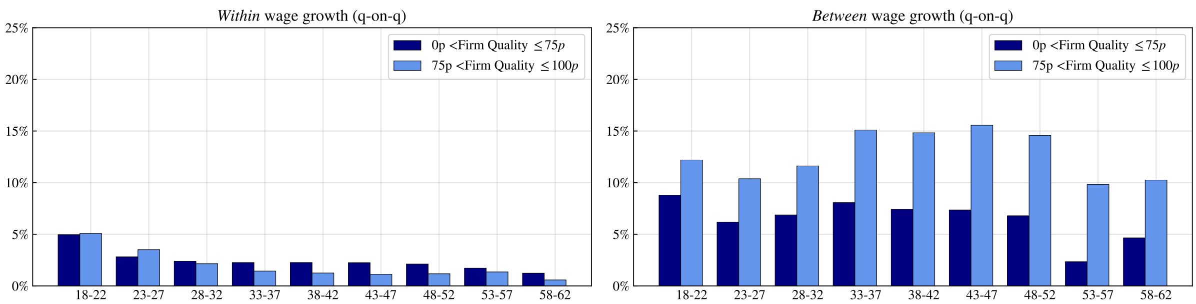
The importance of sorting for human capital accumulation and workers' careers is

**Figure 8.** Search Behavior




**Note:** Search policy function by human capital level and aggregate state, averaged across labor market experience and wage promises.

**Figure 9.** Within vs. Between Wage Growth by Age and Firm Quality in the Model




**Note:** The figure plots the average wage growth, by age and firm quality, *within* employment spells and after employment-to-employment transitions (*between*). For the *betweeen* component, the firm quality quartiles are computed on the distribution of origin firms.

also validated by measuring the correlation of workers ex-post wages with their ex-ante employer quality after an employment to unemployment to employment (EUE) transitions.[20] The correlation is increasing in previous firm qualities, indicating that workers benefit from employment in good firms even once the match is dissolved.[21]

[20]This is an adaptation of Herkenhoff et al. (2018)'s analysis to our model setting. In their paper, they rely on co-workers wages a proxy for firm quality. Given the nature of our framework, we use value-added per employee in the data to measure firm quality, and we control for workers' pre-transition wage.

[21]We report the estimated correlations in the **Table C.1**.

# 5 Aggregate Consequences of Contractions

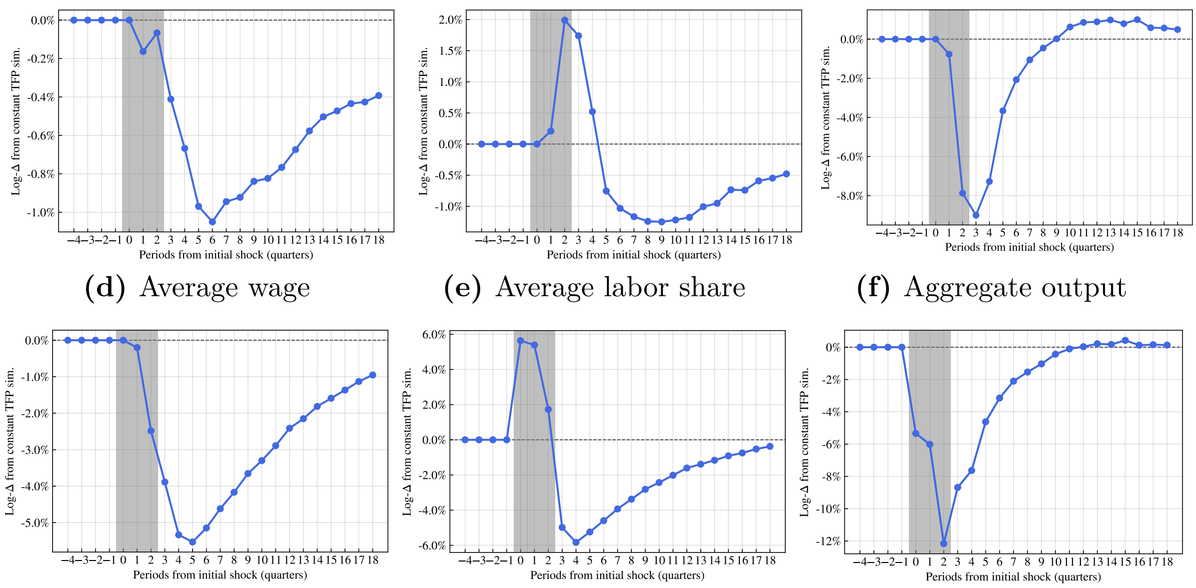
## 5.1 Anatomy of recessions

To compute an impulse response function, we compare a simulation of the model without aggregate shocks and a simulation in which we hit the economy with three consecutive negative realization of the TFP process.[22] We then look at both labor market outcomes of affected cohorts and the response of aggregate GDP to the recession. **Figure 10** reports the main results.

**Figure 10.** Recession Experiment

(a) Firm quality (b) Human capital (c) Employment rate

(d) Average wage (e) Average labor share (f) Aggregate output



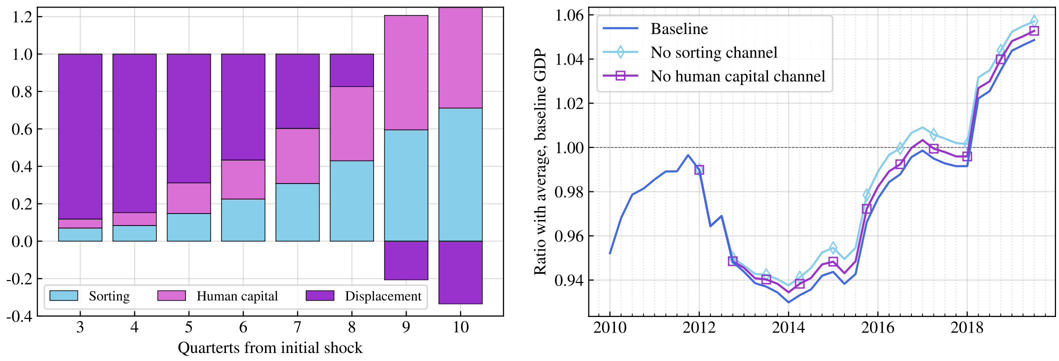

**Note:** The panels in the figure plot the ratios of the aggregate variables between an economy in which we impose a three-quarter, below trend TFP realization and an economy without aggregate shocks, that serves as a benchmark. The shaded area are the quarters in which TFP is below trend.

The dynamics of firm quality and average human capital, respectively in **Figure 10a** and **Figure 10b**, offer a clearer picture of the transmission of aggregate shocks in the economy. While the onset of the recession is accompanied by a sort of "Schumpeterian" response, as implied by the initial marginal increase in human capital shown in **Figure 10b**, firm quality is persistently crippled by the recession, with the average quality of firms active in the economy remaining approximately 0.5% below the no-recession economy even two years after the end of the recession. The average human capital in the long run settles to a similar lower level despite the initial increase. This prolonged reduction in the quality of the factors of production increases the persistence of the initial shock on output beyond the original duration of the recession. In fact, **Figure 10f** shows that even after two years, aggregate output its still approximately

[22] The cumulative drop in TFP is approximately 15% (equally split in each quarter) .

**Figure 11.** Decomposing recessions

**(a)** Decomposing output responses **(b)** The 2011 - 2013 recession in Italy




**Note:** Panel (**a**) shows the relative importance of each transmission channel compared to the baseline recession for the GDP IRF. Panel (**b**) plots the dynamic of the Italian economy with different assumptions on the active channels in the transmission of aggregate shocks to aggregate GDP.

1.5% below its counterfactual level. Significantly, the recession has a significant impact on both the average wage and the average labor share in the economy, as shown in **Figure 10d** and **10e** respectively. As matches that are no longer viable are destroyed, average wages drop less than the average firm output. This allows for a brief spike in the labor share that quickly declines and remains below the counterfactual economy for a long time. The decline occurs for two reasons: the matches that form in this recovery period are still subject to the sullying in firm quality, and the human capital that favors a split in the match surplus increases the firms' share the expense of workers' compensation. This specific dynamic of the labor share also plays a central role in shaping the resilience of the economy to repeated shocks, as we discuss in **Section 5.2**.

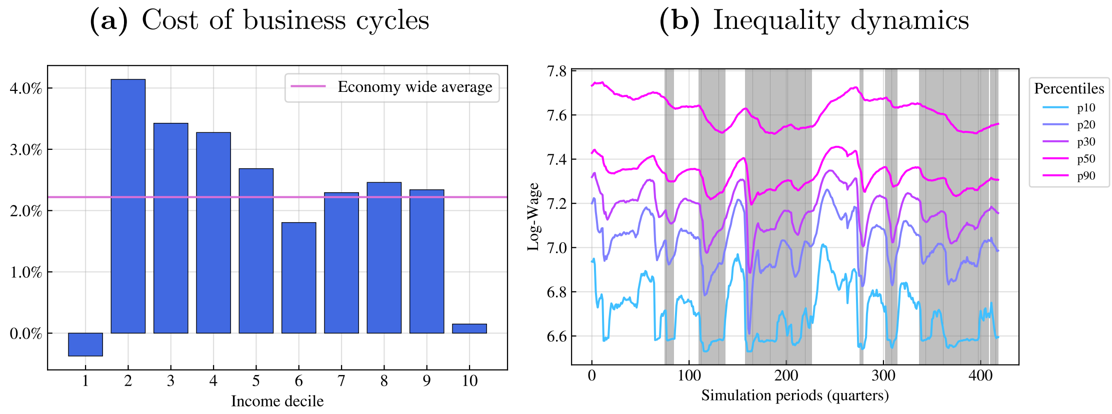
**Decomposing Recessions.** What explains the amplification and persistence of recessionary shocks? Different competing channels are at play. The first, which we call the human capital channel, captures the human capital accumulation that does not take place because of the recessionary event. The second, the sorting channel, amounts to the different joint worker-firm distributions that emerge in the periods following the shock, both because of different search strategies and because of additional unemployment spells. Finally, the standard displacement channel captures the job destruction that takes place because of the negative shock and its spillovers. We decompose the amplification channels of the model economy by shutting down each channel at a time and then comparing the resulting dynamics to the one of a baseline recession. **Figure 11a** decomposes output dynamics after a negative shock as driven by our three channels. The displacement channel is the main driver of the recessionary dynamics on impact, explaining approximately 90% of the initial fall in output. Recovery from displacement, while not immediate because search is frictional, is however relatively fast, in part because unemployed workers have lower reservation wages. We find that in the medium to long run, the sorting and human capital channels become more important, and contribute to the persistence of recessionary events. Interestingly, the three channels play a role in different parts of the workers' cross-sectional distribution (see **Figure 12b**). The displacement channel depends mostly on the size of its left tail: as low-skilled workers are more likely to be separated, the sorting and human capital channels mainly relate to workers in the right tail, for which allocative efficiency and long-run human capital accumulation matter more. This has interesting implications for the fragility of the economy: as the human capital distribution shifts to the right, recessions become less severe on impact, but might become more persistent and have greater long-run effects thought human capital and firm quality match dynamics.

**Decomposing the 2011–2013 recessions in Italy.** To quantify the importance of each channel in the data, we construct an experiment with the Italian business cycle. We use realizations of our shock process that matches the evolution of the Italian TFP, as in **Figure 6**, and we run three counterfactual simulations: 1) the 2011–2013 recession happens but there is no cyclical effect on displacements, 2) the recession happens but the sorting channel during the recession is muted, and 3) the human capital accumulation channel during the recession is muted. Therefore, the dynamics of the second and third scenarios represent the dynamics of the Italian economy if the recession had no impact on workers' search behavior or human capital accumulation. **Figure 11b** shows the baseline evolution of the cyclical component of the Italian economy, as well as the three counterfactuals simulations. When the sorting channel is muted, the economy recovers faster, but most importantly enters the following expansion with a greater ability to capitalize on growth: that factor alone would account for almost a 1.5% difference in the level of GDP cycle as of 2019. Silencing the human capital loss induced by the recession has a quantitatively similar effect. Compounding the two, they amount for more than a 3% difference from baseline GDP levels. Policies addressing distortions in human capital accumulation and worker-firm sorting can thus play a large role in reducing the hysteretic effects of recessions.

## 5.2 Inequality and business cycles

**Costs of Business Cycles.** Influential work by Lucas (1987) argues that welfare gains from reducing business cycle volatility are negligible, and quantifies them in less than 0.1 percentage points of consumption-equivalent units. By doing an analogous calculation with our model, we estimate the cost of business cycles to be, on average, greater than 2 percentage points, more than two orders of magnitude above the Lucas estimate. Our estimate is quantitatively close to the one in Barlevy (2004), who first observed the potentially sullying effects of recessions. The richness in heterogeneity in our model though

**Figure 12.** Business Cycle Costs and Inequality



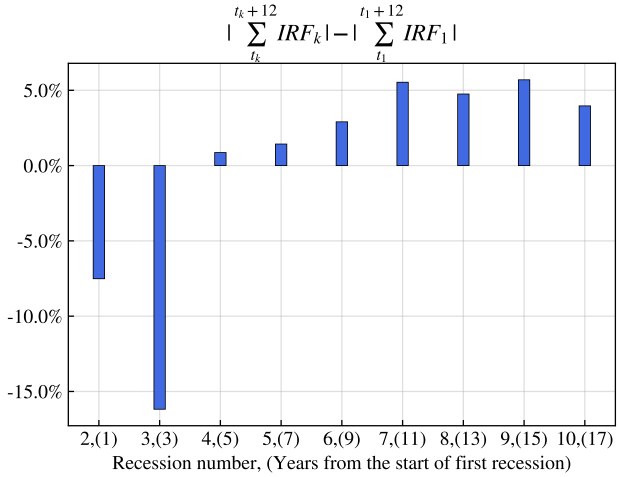

**Note:** Panel (**a**) plots the reduction in consumption-equivalent utility due to aggregate fluctuations; Panel (**b**) reports the dynamics of income for different percentiles of the income distribution.

allows us to estimate how the welfare costs of business cycles vary along the income distribution. We can thus decisively show how welfare costs of business cycles crucially interact with income inequality. **Figure 12a** plots the cost of business cycles for income deciles. What emerges is that the extreme deciles, the first and the tenth, bear very little to no costs from business cycle fluctuations. For different reasons, both deciles are less affected by separation risk due to aggregate fluctuations: while the bottom decile mostly comprises unemployed workers, the top decile includes workers whose job tenure is robust to recessions. The welfare costs of business cycles are, however, much larger for all intermediate deciles, and have an interesting U-shape. Up until the seventh decile, the costs of aggregate fluctuations decrease with income: lower incomes are associated with more-fragile jobs and to a higher likelihood of unemployment in recessions. For the last three deciles, costs of business cycles are instead associated with long-run impacts on careers. For workers closer to the top of the income distribution, recessions can have strong negative effects, as the deterioration in sorting and the ensuing lower human capital accumulation prevents them from getting to their best possible employment.

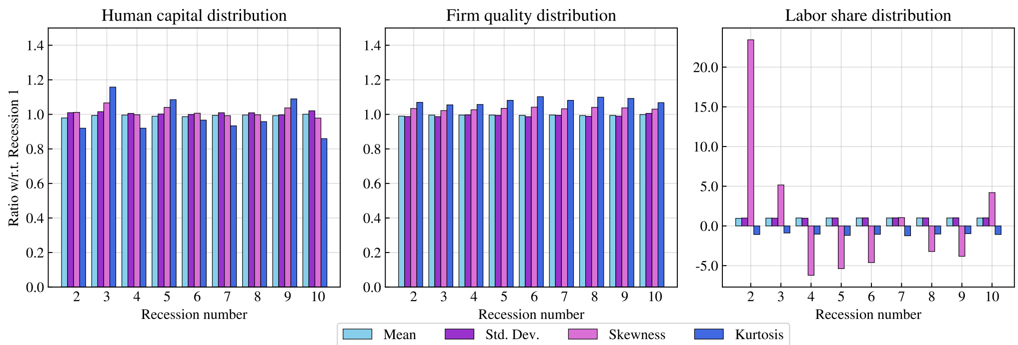
**Inequality Dynamics.** Heathcote, Perri and Violante (2020) show that recessions impact inequality with persistence, affecting the earnings of workers in the left tail of the income distribution the most. **Figure 12b** illustrates the dynamics of inequality around business cycles by displaying the pattern of losses across the earnings distribution. Recessions hit the poorest workers the hardest, and worsening job prospects push some of them out of the labor force. A prediction of the model that departs from existing literature is that the persistence of earnings losses varies across the distribution: while workers with less human capital display more volatility in earnings (mostly due to displacement, see **Figure 4**), the impact on workers with high human capital is dampened but quite persistent.

**Figure 13.** State dependence and Business Cycles

**(a)** Cumulative losses




**(b)** Match-related distributions




**Note:** The figure plots a comparison between a series of simulations with two recessionary shocks of the same magnitude, one successive the other but at different horizons. The top panel shows the cumulative difference between the two recessions. The bottom panels report the ratios of the first four moments of selected variables in the quarter before each recession hit the economy. For skewness and kurtosis the denominator is the absolute value of the first recession moment.

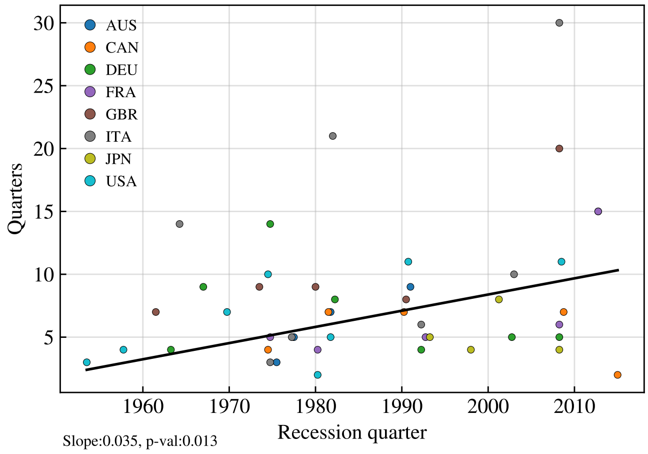
**Double Dips and Long Expansions** The model economy exhibits significant state dependence and can thus speak to the different impacts of shocks depending on the state of the economy when the shock hits. An important question is whether the depth of recessions is increasing in either built-in fragility from preceding long expansions or from the misallocating effects of a previous recession. To understand this, we perform a “double recession” experiment. The experiment works as follows: at time 0, we hit the economy with a recession identical to the one discussed in **Section 5.1**, then we track the evolution of human capital, firm quality, and labor share by collecting the first four moments of their distributions, absent other aggregate shocks. The three bottom panels in **Figure 13** report their values for nine different points in time following the first recession, starting one year after the return of aggregate shocks to trend and then every two years for the following eight. Starting from the cross-sectional distribution computed this way, we hit the economy again with an identical recession, separately at each time horizon specified above. The top panel in **Figure 13** reports the difference of cumulated output reponses between the baseline and the "second" recession. The first two recessions, those that happen within five years from the first, have smaller cumulated losses. Fragility builds up over time, and recessions become more severe, peaking at the 11-year interval between the two shocks. This rather surprising result can be interpreted in light of the first decomposition of recessions, which shows the initial severity depends mostly on the impact of displacements: when the economy is recovering from a previous shock, even if unemployment has been re-absorbed, the labor share is unusually skewed to the left. This means that more workers have low wages compared to their baseline status, making the leverage problem in their match less severe. Similarly, when double dips hurt the most (for recessions number 7, 8, and 9) two facts emerge: the human capital distribution tails are less flat, but the labor share tails are fatter. This imbalance results in more separations and a more severe recession. The average length of an expansion in the data is about six years. Our results then point to the building up of fragility coming from longer-than-average expansions. In **Appendix C**, we show suggestive evidence in favor of this mechanism. In absence of more micro data on other economies, however, we do not have enough data points to reliably test it. While more research is definitely needed on this specific issue, we believe this result points to the usefulness of tracing the evolution of labor markets' micro-dynamics to highlight potential fragilities of the economy.

### 5.3 What accounts for the increased length of recessions?

The time economies take to recover from recessions has increased across developed economies in the last thirty years. **Figure 14** shows the average number of quarters aggregate GDP has been below trend during recessions for a subset of advanced economies. From the mid-1980s and early 1990s, in particular, this measure has consistently increased.[23] This is consistent with evidence presented by Fukui, Nakamura and Steinsson (2023) on the slower recovery of employment after recessions. Among other factors, an increase in job polarization and the rise in the skill premium are contemporaneous phenomena with this rise in the time economies need to recover from recessions (Goldin and Katz 2007; Goos, Manning and Salomons 2009). Our model provides a useful structure to check whether human capital accumulation and the

[23]Specifically, in the data, we define a recession as occurring after two consecutive quarters of negative GDP growth, and within each recession, we count the number of quarters in which GDP realizations are below trend. We obtain a similar picture if we look at alternative definitions of recession lengths, such as the number of quarters that are needed to reach pre-recession GDP levels and the number of consecutive quarters with negative GDP growth. However, these definitions cannot be transferred directly to model simulations as they rely on measures of GDP growth, which we do not explicitly model. Therefore, in our simulations, the number of consecutive quarters GDP remains below trend is our preferred measure of recession lengths.

**Figure 14.** Length of Recessions Over Time




**Note:** The figure reports the average duration of recessions for a set of OECD countries. Specifically, for each recession, we compute the number of quarters each economy's GDP remains below trend.

sorting dynamics in the labor market contribute to these aggregate developments. Our model is calibrated to match the differences in career paths and human capital accumulation of graduates and non-graduates in Italy in the last ten years. Our model shows that college-educated workers' human capital accumulates faster than other workers'. The widening of this gap through a higher weight of firm quality in the human capital production function (human capital deepening) proxies the development of a skill premium over time. We conduct the following experiment to analyze whether the rise in human capital's deepening and the lengthening of job ladders have contributed to the increase in length of recoveries. We consider two simulations of the model: our baseline and a counterfactual simulation in which the accumulation of human capital is the same across education levels. In the counterfactual economy the average earnings' growth for graduates is only 3% higher than non-graduates.[24] In our baseline economy graduates enjoy a faster human capital accumulation, which translates to approximately 30% higher earnings for graduates than non-graduates, on average.

The presence of a high-skill premium leads to recessions that are approximately 21% longer than those occurring without skill premium. In relative terms, the change in the length of recessions is remarkably close to what is observed for the subset of advanced economies in **Figure 14**. For these countries, in fact, the average duration of below-trend GDP realizations in recessions increases by approximately 29% comparing the periods before and after 1990.[25]

[24]Notice that graduates, while being older at labor market entry, draw their initial human capital from a distribution that dominates stochastically to a first order of the distribution of non-graduates. Therefore, starting from higher initial earnings, the model predicts some small further divergence in income.

[25]For Italy, the increase in this measure has been slightly more pronounced. Recession lengths went from 11 to 15 quarters, approximately a 36% increase.

We also test competing hypotheses, related to concurrent long-term trends, to show how they relate to recoveries' lengths. Increasing the firm-quality elasticity in the production function by approximately 6%, similar to the change in the share of labor compensation over GDP in Italy after 1990, would not lead to any change in the length of recessions. Jaimovich and Siu (2009) argue that population aging has had a significant impact on the business cycle properties advanced economies. To see how this has affected recession lengths, we simulate the model without a skill premium using demographic weights from the 1970 and 2011 Italian censuses, and we calculate the average recession length in these counterfactual scenarios. We find that accounting only for these demographic changes results in a 14% increase in the average recession length, about half of what observed in the data.[26]

# 6 Policies and the Business Cycle

We showed in the previous sections that business cycles are amplified by contracting frictions, search frictions, and allocative inefficiencies. Because recessions generate persistent welfare losses, there is a natural role for policy. In this section, we discuss the role of two specific policy interventions using our model environment as a laboratory.

**Minimum Wage.** The rationale for minimum wages in our model is simple. Beyond its redistributive nature, minimum wages can induce a flatter job ladder for younger workers, potentially increasing the subsequent accumulation in human capital and making the economy less fragile to fluctuations.[27] A potential cost of minimum wages is the de facto disappearance of low-paid jobs that effectively act as stepping stones. The equilibrium effect is thus ambiguous, and the overall effects might depend on the level of the minimum wage. We impose a minimum wage equal to the long run median of entry wages from unemployment, which is equivalent to approximately €880 per month.

**Cyclical Transfers.** We analyze the effect of linking the amount of unemployment benefits to the aggregate state of the economy (a countercyclical unemployment benefit). Increasing the unmployment benefit when it is most needed is an immediate way to prevent incomes from falling in recessions. [28] In our setting part of the cost of increasing assistance to workers in recessions is implicitly sustained by decreasing the generosity of

[26] If we were to consider both channels simultaneously it would be sufficient to increase the skill premium so that earnings for graduates are approximately 24% higher than non-graduates to obtain a relative increase in recession lengths that matches exactly what is observed in the data.

[27] Dustmann et al. (2022), for instance, show evidence of a similar mechanism in Germany, where a reform that increased the minimum wage helped workers sort into higher-quality jobs

[28] A similar kind of policy has been used extensively during the Covid recession, in departure from more traditional, means-tested interventions. The US government, for instance, spent $169 million in 2020 and $402 million in 2021 in one-time stimulus payments and $119 million in 2020 and $206 million in 2021 in unemployment benefits (including $300 per week supplement until September 2021).

benefits in expansionary periods. We calibrate transfers to be increasing in the severity of recessions so that their correlation with aggregate productivity equals $-1.5$. This implies that, in the lowest possible realization, transfers can go up to €980 per month.[29]

**Discussion.** **Table 4** presents outcomes of our preferred calibration for each of the policy experiments are presented. The minimum wage, while effective in affecting the search behavior of more fragile workers, severely impacts job creation. This is reflected in an additional 5.7% of workers leaving the labor force, and in an additional 0.6% of unemployed workers. As a result, employment falls by more than 6%. The additional fall in output levels can be attributed to the lower human capital accumulation caused by longer unemployment spells on average. These effects get in the way of minimum wages helping workers build resilience in the face of output fluctuations, and the cost of business cycles as a share of consumption grows substantially. The combined impact on unemployment and labor force participation offsets gains in earnings inequality as measured by the Gini index: the estimated zero impact of the policy captures an increase in inequality due to a drop in labor earnings on the extensive margin, together with a compression of the wage distribution, conditional on workers being employed. The discussed effects are amplified for higher levels of the minimum wage, mostly due to a growing impact on job creation. On the other hand, reducing the level of minimum wages gradually makes it less binding, until no output effects are detectable. In no calibration we do find that output levels grow or overall inequality levels decrease using minimum wages using our estimated model.

**Table 4.** Impact of Policy Experiments

| | Scenario | | |
|---|---|---|---|
| | Baseline | Minimum Wage | Cyclical Transfers |
| Output Level | 1.0* | 0.9 | 1.09 |
| Welfare | 1.0* | 0.89 | 1.07 |
| Unemployment | 9.1% | 9.7% | 9.5% |
| Employment | 62.6% | 56.3% | 71.5% |
| Sorting | 0.55 | 0.62 | 0.58 |
| Gini Index | 0.15 | 0.15 | 0.16 |
| Search | 1.0* | 1.05 | 1.02 |

**Note**: Model estimates. For outcomes with an *: baseline values are normalized to 1.

The transfer policy increases welfare by providing broad insurance against the fundamental recession-related risk, that is the fall in earnings due to unemployment. In

[29] In the baseline simulation, transfers are allowed to increase up to €895. Because of the nature of our model, we are abstracting from considerations on how the policy is financed. The overall cost of imposing countercyclical transfers is around 4.5% of GDP, which is likely to significantly dampen the reported gains on output and welfare.

addition, it increases overall employment because transfers require workers to be active in the labor force. Output then increases both because more workers are active, and because the overall sorting in the economy improves. Our analysis yields several observations on the impact of policies on sorting. The sorting measure is higher than in the baseline model both when minimum wage is introduced or when transfers are implemented. In the first case, the increase in sorting does not reflect an improved allocative efficiency of the economy but simply the moving into inactivity of part of the workers on the left tail of the human capital distribution. In the second case, sorting is higher because there are more good workers in good firms. Consistent with Song et al. (2019), increases in sorting generate more inequality, in our case captured by a small increase in the Gini index.

# 7 Conclusions

We have set up a new and tractable model of on-the-job search and human capital accumulation that features heterogeneity both on the worker side and the firm side. Ex-ante heterogeneous workers accumulate on-the-job experience which augments their skills and moves them up in the job ladder. Consistent with the data, rigidity in labor costs amplifies negative shocks to firms, and generates inefficient separations. We establish that workers that look for employment in bad economic times direct their search toward less-productive firms. Search frictions and aggregate uncertainty prevent an efficient allocation of workers to firms and expose different cohorts of workers to different human capital accumulation paths depending on the aggregate state at the time of entry to the labor force. Limits on workers' ability to accumulate human capital impose a drag on the overall labor productivity of the economy after recessions that tend to persist for a long time. Alterations to the sorting induced by recessions are slow to reverse and contribute not only to slow recoveries but also to long-run changes in the structure of the economy.

We show that aggregate fluctuations interact with shifts in the cross-sectional composition of labor markets, in ways consistent with the existence of hysteretic effects of recessions. We use the model to shed light on two open questions in business cycle literature. We show that increases in the importance of firms for workers' human capital accumulation, together with aging, can explain the increased length of recessions in recent decades. We also find that distributional channels can explain the reason why subsequent recessions are more or less severe when the economy is hit by shocks within years after a first recession. The framework lends itself naturally to be used as a laboratory for policy experiments. Our experiments support the idea that increasing the generosity of unemployment transfers in severe recessions can improve the overall allocative efficiency of the economy in the long run.

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# Appendices

## A Existence of a Block Recursive Equilibrium

In order to show that a Block Recursive Equilibrium (BRE) exists in our model we need to show that the equilibrium contracts, the workers' and the entrepreneurs value and policy functions do not depend on the distribution of employed and unemployed workers. This implies that the only element of the aggregate state that matters for a firm when making an hiring decision is the state of aggregate productivity but not the distribution of worker types (e.g. employed vs unemployed).

**Proposition A.1.** *A Block Recursive Equilibrium as defined in Definition 2.3 exists.*

*Proof.* We follow the approach in Menzio, Telyukova and Visschers (2016), Herkenhoff, Phillips and Cohen-Cole (2019) and prove the existence of a BRE using backward induction.

Consider the lifetime values of an unemployed and an employed worker before the production stage in the last period of households lives with $\tau = T$ :

$$U(h, T, \iota; \Omega) = u(b(h, T)) \tag{A.1}$$

$$V(h, T, \iota; \Omega) = u(w(a)), \tag{A.2}$$

their values trivially do not depend on the distribution of types as both valuations are 0 from $T+1$ onward. Hence, $U(h, T, \iota; \Omega) = U(h, T, \iota; a)$ and $V(h, T, \iota; \Omega) = V(h, T, \iota; a)$.

The optimal contract for agents aged $\tau = T$, instead, solves the following problem

$$J(h, T, W, y; \Omega) = \sup_{w} [f(y, h; a) - w] \quad s.t.\ W = u(w),$$

that clearly does not depend on the distribution of worker types due to the directed search protocol and where the aggregate state only affects the promised utility and the optimal wage through realization of the aggregate productivity processes. Therefore, $J(h, T, \iota, W, y; \Omega) = J(h, T, \iota, W, y; a)$.

This also implies that the equilibrium market tightness

$$\theta(h, T, \iota, W; \Omega) = q^{-1}\left(\frac{c(y)}{J(h, T, \iota, W, y; a)}\right)$$

is independent from the distribution of worker types and it is only affected by realization of aggregate productivity, so $\theta(h, T, W; a)$.

This in turn implies that the search problem workers face at the beginning of the last period of their lives depends on the aggregate state only through aggregate productivity $a$:

$$R(h, T, \iota, V; a) = \sup_v \Big[p(\theta(h, T, \iota, v; a))\big[v - V\big]\Big],$$

does not depend on the distribution of worker types.

Stepping back at $\tau = T-1$, the value functions for the unemployed and the employed agents are solutions to the following dynamic programs

$$\sup_v u(b(h, T-1)) + \beta \mathbb{E}_{\Omega,\psi}\Big(U(h', T, \iota; a') + p(\theta(h, T, \iota, v; a'))\big[v - U(h', T, \iota, ; a')\big]\Big)$$

$$u(w) + \beta \mathbb{E}_{\Omega,\psi}\left(\begin{array}{c} \lambda U(h', T, \iota; a') + \beta(1-\lambda)W + \\ +\beta(1-\lambda)\lambda_e \max(0, R(h', T, \iota, , W); a')]\Big] \end{array}\right),$$

where both do not depend on the distribution of worker types.

The optimal contract at this step is a solution to

$$\begin{aligned} J_t(h, T-1, \iota, V, y; a) = \sup_{\{\pi_i, w_i, W_i\}} \sum_{i=1,2} \pi_i \Big( & f(y, h; a) - w_i \\ & + \mathbb{E}_{\Omega,\psi}\left[\widetilde{p}(h', T, W_{i,\Omega'}; a')(J(h', T, y, W_i; a')\right] \Big) \end{aligned}$$

$$\begin{aligned} s.t.\ V = \sum_{i=1,2} \pi_i \left(u(w_i) + \mathbb{E}_{\Omega,\psi}\widetilde{r}(h', T, W_i; a')\right),\ & h' = \phi(h, y, \iota, \psi) \\ \mathbb{E}_{\Omega,\psi} \sum_{i=1,2} \pi_i \left(\mathbb{E}_{\Omega,\psi} J(h', T, \iota, W_i, y; a')\right) \geq 0 \text{ and } & t \leq T \end{aligned}$$

which does not depend on types distribution.

Therefore, also the equilibrium tightness and the search gain at $T-1$ are independent from types' distributions, as

$$\theta(h, T-1, \iota, W; a) = q^{-1}\left(\frac{c(y)}{J(h, T-1, \iota, W, y; a)}\right)$$
$$R(h, T-1, \iota, V; a) = \sup_v \Big[p(\theta(h, T-1, \iota, v; a))\big[v - V\big]\Big].$$

Stepping back from $\tau = T-1, ..., 1$ and repeating the arguments above completes the proof. □

# B Proofs of model properties

For compactness of notation, we omit the dependence on education level, which is a fixed characteristic, and the idiosyncratic human capital shock, which is additive, from the proofs in Appendices. The logic of the proofs follows without loss of generality. We reserve a more detailed discussion of the model theoretical properties in the Online Appendix.

## B.1 Proof of Property 3.1

*Proof.* Note: throughout the proof we drop the dependence of the functions to the state $(h, \tau, \Omega)$ to ease readability.

If the function $f_v$ is an injective function then it defines a one-to-one mapping between $\mathcal{Y}$ and $\mathcal{V}$ so that for $(y_1, y_2) \in \mathcal{Y}$, and $f_v(y_1) = W_1$ and $f_v(y_2) = W_2$, $(W_1, W_2) \in \mathcal{V}$, $f_v(y_1) = f_v(y_2) \Rightarrow y_1 = y_2$.[30] We proceed by contradiction. To begin, assume that $f_v(y_1) = f_v(y_2)$ and $y_1 \neq y_2$.

As the optimal contract is a concave function in firm quality, we know that the tangents at each point are above the graph of the function. Thus, we can define the tangents at the two points $y_1, y_2$ as

$$T_1(y) \equiv J(y_1) + \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_1}(y - y_1) \quad \text{and} \quad T_2(y) \equiv J(y_2) + \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_2}(y - y_2).$$

Without loss of generality, consider the case in which $y_2 > y_1$. Knowing that $T_i(y) \geq J(y)$ for $i = 1, 2$ due to the concavity of $J(\cdot)$, we can define the following inequalities:

$$T_1(y_2) - J(y_2) \geq 0 \quad \text{and} \quad T_2(y_1) - J(y_1) \geq 0.$$

[30] As the contrapositive of Definition 2.2 in Rudin (1976), that defines a one-to-one mapping for $(x_1, x_2) \in A$ as $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

Using the definitions for the tangents at $y_1$ and $y_2$ they imply that

$$\frac{J(y_2)-J(y_1)}{y_2-y_1} \leq \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_1} \quad \text{and} \quad \frac{J(y_2)-J(y_1)}{y_2-y_1} \geq \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_2},$$

hence combining the inequalities we get that

$$\left.\frac{\partial J(y)}{\partial y}\right|_{y=y_2} \leq \frac{J(y_2)-J(y_1)}{y_2-y_1} \leq \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_1}. \tag{B.1}$$

However, the free-entry condition in vacancy posting implies that in the submarket $(h,\tau,W)$ both firms must be respecting $c(y_i) = q(\theta)\beta J(y_i)$ for $i = 1,2$. As $c(y_i)$ is a linear function of firm quality $\frac{\partial c(y_i)}{\partial y_i} = c$ for $i = 1,2$ and therefore from the free-entry condition:

$$c = q(\theta)\beta \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_i}$$

which is a contradiction of the slopes of the two tangents being decreasing as shown in Equation (B.1). Note that if $c(y)$ is convex and twice differentiable, then the derivatives of $c(y)$ are increasing in $y$ while the derivatives of $J(\cdot)$ are decreasing leading again to a contradiction. The proof for the case in which $y_1 > y_2$ follows the same arguments and leads to a similar contradiction on the implied slopes of the optimal contract and those implied by the free entry condition. □

## B.2 Proof of Properties 3.2 and 3.4

The following propositions characterize the properties of workers' optimal search strategies that solve the search problem in (9). We start proving a useful lemma and then provide the proofs for the two properties.

**Lemma B.1.** *The composite function $p(\theta(h,\tau,v;\Omega))$ is strictly decreasing and strictly concave in $v$.*

*Proof.* For this proof we follow closely Menzio and Shi (2010), Lemma 4.1 (ii). From the properties of the matching function we know that $p(\theta)$ is increasing and concave in $\theta$, while $q(\theta)$ is decreasing and convex. Consider that the equilibrium definition of $\theta(\cdot)$ is

$$\theta(h,\tau,v;\Omega) = q^{-1}\left(\frac{c(y)}{J(h,\tau,v,y;\Omega)}\right),$$

and that the first order condition for the wage and the envelope condition on $V$ of the optimal contract problem in (5) implies

$$\frac{\partial J(h,\tau,v,y;\Omega)}{\partial v} = -\frac{1}{u'(w)}.$$

so that as $u'(\cdot) > 0$, $J(\cdot)$ is decreasing in $v$.

From the equilibrium definition of $\theta(\cdot)$ and noting that $q^{-1}(\cdot)$ is also decreasing due to the properties of the matching function we have that

$$\frac{\partial \theta(h,\tau,v;\Omega)}{\partial v} = \frac{\partial q^{-1}(\xi)}{\partial \xi}\bigg|_{\xi=\frac{c(y)}{J(h,\tau,v,y;\Omega)}} \cdot \left(-\frac{\partial J(h,\tau,v,y;\Omega)}{\partial v}\right) \cdot \frac{c(y)}{(J(h,\tau,v,y;\Omega))^2} < 0,$$

which, in turn, implies that

$$\frac{\partial p(\theta(h,\tau,v;\Omega))}{\partial v} = \frac{\partial p(\theta)}{\partial \theta}\bigg|_{\theta=\theta(h,\tau,v;\Omega)} \cdot \frac{\partial \theta(h,\tau,v;\Omega)}{\partial v} < 0.$$

Suppressing dependence on the states $(h,\tau,y,\Omega)$ for readability, to prove that $p(\theta(v))$ is concave, consider that $J(v)$ is concave[31] and a generic function $\frac{c}{v}$ is strictly convex in $v$. This implies that with $\alpha \in [0,1]$ and $v_1, v_2 \in \mathcal{V}$, $v_1 \neq v_2$:

$$\frac{c}{J(\alpha v_1 + (1-\alpha)v_2)} \leq \frac{c}{\alpha J(v_1) + (1-\alpha)J(v_2)} < \alpha\frac{c}{J(v_1)} + (1-\alpha)\frac{c}{J(v_2)}.$$

As $p(q^{-1}(\cdot))$ is strictly decreasing the inequality implies that

$$\begin{aligned} p\left(q^{-1}\left(\tfrac{c}{J(\alpha v_1+(1-\alpha)v_2)}\right)\right) &\geq p\left(q^{-1}\left(\tfrac{c}{\alpha J(v_1)+(1-\alpha)J(v_2)}\right)\right) \\ &> \alpha p\left(q^{-1}\left(\tfrac{c}{J(v_1)}\right)\right) + (1-\alpha)p\left(q^{-1}\left(\tfrac{c}{J(v_2)}\right)\right), \end{aligned}$$

and as $\theta(v) = q^{-1}(\frac{c}{J(v)})$:

$$p(\theta(\alpha v_1 + (1-\alpha)v_2)) > \alpha p(\theta(v_1)) + (1-\alpha)p(\theta(v_2))$$

so that $p(\theta(v))$ is strictly concave in $v$. □

The proofs for the two properties are as follows:

*Proof.* The proofs follow closely Shi (2009), Lemma 3.1 and Menzio and Shi (2010), Corollary 4.4. More formally, for each triplet $(h,\tau,\Omega)$ given at each search stage, we can re-define the search objective function as $K(v,V) = p(\theta(v))(v-V)$ and $v^*(V) \in \arg\max_v K(v,V)$ as the function that maximises the search returns (i.e. the optimal search strategy of the worker) and prove the following.

We first show that $K(v,V)$ is strictly concave in $v$. Consider two values for $v$, $v_1, v_2 \in \mathcal{V}$

[31] $J(\cdot)$ concave give the two-point lottery in the structure of the contract. See Menzio and Shi (2010) Lemma F.1.

such that $v_2 > v_1$ and define $v_\alpha = \alpha v_1 + (1-\alpha)v_2$ for $\alpha \in [0,1]$. Then by definition:

$$\begin{aligned}
K(v_\alpha, V) &= p(\theta(v_\alpha))(v_\alpha - V) \\
&\geq [\alpha p(\theta(v_1)) + (1-\alpha)p(\theta(v_2))][\alpha(v_1 - V) + (1-\alpha)(v_2 - V)] \\
&= \alpha K(v_1, V) + (1-\alpha)K(v_2, V) + \alpha(1-\alpha)[(p(\theta(v_1)) - p(\theta(v_2))](v_2 - v_1) \\
&> \alpha K(v_1, V) + (1-\alpha)K(v_2, V)
\end{aligned}$$

where the first inequality follows from the concavity of $p(\theta(\cdot))$ (this is true if $J(\cdot)$ concave with respect to $V$) and the second inequality stems from the fact that $p(\theta(\cdot))$ is strictly decreasing hence $\alpha(1-\alpha)[(p(\theta(v_1)) - p(\theta(v_2))](v_2 - v_1) > 0$.

**Weakly increasing in promised utility.** Consider a worker employed in a job that gives lifetime utility $V$. Given that $v \in [\underline{v}, \overline{v}]$, and that submarkets are going to open depending on realizations of the aggregate productivity, $a$, there is only one region in the set of promised utilities where the search gain is positive. This set is $[V, v(a)]$ with $v(a)$ being the highest possible offer that a firm makes in the submarket for the worker $(h, \tau)$. Any submarket that promises higher than $v(a)$ is going to have zero tightness. Therefore, the optimal search strategy for $V \geq v(a)$ is $v^*(V) = V$, as $K(V, v(a)) = K(V, V) = K(\overline{v}, V) = 0$ (the search gain is null given the current lifetime utility $V$). For $V \in [V, v(a)]$, instead, as $K(v, V)$ is bounded and continuous, the solution $v^*(V)$ has to be interior and therefore respect the following first order condition

$$V = v^*(V) + \frac{p(\theta(v^*(V)))}{p'(\theta(v^*(V)) \cdot \theta'(v^*(V))}. \tag{B.2}$$

Now consider two arbitrary values $V_1$ and $V_2$, $V_1 < V_2 < \overline{v}$ and their associated solutions $W_i = v^*(V_i)$ for $i = 1, 2$. Then, $V_1$ and $V_2$ have to generate two different values for the right-hand side of (B.2). Hence, $v^*(V_1) \cap v^*(V_2) = \emptyset$ when $V_1 \neq V_2$. This also implies that the search gain evaluated at the optimal search strategy is higher than the gain at any other arbitrary strategy so that $K(W_i, V_i) > K(W_j, V_i)$ for $i \neq j$. This implies that

$$\begin{aligned}
0 >& [K(W_2, V_1) - K(W_1, V_1)] + [K(W_1, V_2) - K(W_2, V_2)] \\
&= (p(\theta(W_2)) - p(\theta(W_1)))(V_2 - V_1),
\end{aligned}$$

thus, $p(\theta(W_2)) < p(\theta(W_1))$. As $p(\theta(\cdot))$ is strictly decreasing (see Lemma B.1), then $v^*(V_1) < v^*(V_2)$. Uniqueness follows directly from strict concavity.

**Decreasing in $a$.** We are now interested in the sign of:[32]

$$\frac{\partial \bar{J}}{\partial a}\bigg|_{W_y,\pi,w,\{W'_y\}} = \frac{\partial f(\cdot)}{\partial a} + \beta \mathbb{E}_\Omega \left[ \frac{\partial \tilde{p}(\cdot)}{\partial a}\bigg|_{W_y,\pi_i,w,\{W'_y\}} \bar{J}' \right].$$

Now notice that, in equilibrium,

$$\frac{\partial \widetilde{p}(\theta)}{\partial a} \propto -\frac{\partial p(\theta)}{\partial a} = \underbrace{\frac{\partial p(\theta)}{\partial \theta}}_{>0} \cdot \underbrace{\frac{\partial \theta}{\partial J(\cdot)}}_{>0} \cdot \frac{\partial J(\cdot)}{\partial a}$$

where the sign of the second derivative on the right hand side comes from the free entry condition and the properties of vacancy filling probability function $q(\cdot)$. Given this, it has to be that $\frac{\partial p(\theta)}{\partial a}$ and $\frac{\partial J(\cdot)}{\partial a}$ have the same sign in equilibrium. Now, if both are strictly positive, both statements of our proposition are immediately true. Let's now assume they are both negative or zero. If this is the case, then $\frac{\partial \tilde{p}(\cdot)}{\partial a} \geq 0$. But this implies $\frac{\partial \bar{J}}{\partial a} > 0$, which is a contradiction. □

## B.3 Proof of Property 3.3

The proof follows from the characteristic of workers optimal behaviour (see 3.2) and the following proposition.

**Proposition B.1.** *The Pareto frontier $J(h, \tau, \iota, W, y; a, \mu)$ is increasing in the aggregate productivity shock $a$.*

*Proof.* We proceed by backward induction.[33] For workers in period $T$, given that the firm increases its production while keeping the worker at least indifferent, $J$ is at least weakly increasing in $a$. However, the firm can also feasibly increase the worker's wage by $\varepsilon$, with $\varepsilon < \frac{\partial f(\cdot)}{\partial a}$. $J$ is thus strictly increasing in $y$. Consider now a worker who is $T-1$ periods old. A firm matched to a worker in submarket $\{h, T-1, y, W_{y,\Omega}\}$ will face the following Pareto frontier

$$\begin{aligned} J(h, T-1, y, W_y; a, \mu) = \sup_{w, W'_y} \Big( & f(h, y; a) - w \\ & + \mathbb{E}_\Omega \left[ \widetilde{p}(h', T, W'_y; a', \mu')(f(h', y; a') - w') \right] \Big). \end{aligned}$$

Assume that aggregate productivity increases from $\bar{a}$ to $\bar{a} + \varepsilon$ and that the firm keeps its policies constant. We aim at proving that, even in such a case, firm value increases

[32] We assume that $J' = f(h', y; a') - w'$ is constant with respect to $a$. It is possible to prove, by backward induction, that this assumption is without loos of generality for the sake of the proof.

[33] For compactness of notation, we omit without loss of generality the two-point lottery in the equations in the proof.

while keeping the worker at least indifferent. If this is the case, it is *a fortiori* true that $J$ increases in $a$ after reoptimizing firms' policies. $\square$

## B.4 Proof of Property 3.5

*Proof.* Consider the firm problem in Equation (5). For $i = 1, 2$, the first order conditions with respect to the wage and the promised utilities are:

$$[w_i] : \ \lambda = \frac{1}{u'(w_i)} \tag{B.3}$$

$$[W_i] : \ \frac{\partial \widetilde{p}(\cdot)}{\partial W_i} J(\cdot) + \widetilde{p}(\cdot) \frac{\partial J(\cdot)}{\partial W_i} + \lambda \frac{\partial \widetilde{r}(\cdot)}{\partial W_i} = 0. \tag{B.4}$$

Note that by definition,

$$\widetilde{r}(h, \tau, V; \Omega) \equiv \lambda U(h, \tau; \Omega) + (1 - \lambda) \Big[ W + \lambda_e \max\{0, R(h, \tau, V; \Omega)\} \Big]$$

therefore we can use the envelope theorem as in Benveniste and Scheinkman (1979), Theorem 1 and the definition in Equation (12) to derive an expression for the derivative of the employment value in $t + 1$ as the period ahead of the following:

$$\frac{\partial \widetilde{r}(h, \tau, W; \Omega)}{\partial W} = \widetilde{p}(h, \tau, W; \Omega).$$

Similarly, using the envelope condition on the firm problem and the first order condition for the wage, we can establish that

$$\frac{\partial J(h, \tau, y, W; \Omega))}{\partial W} = -\lambda \ \therefore \ \frac{\partial J(h, \tau, W, y; \Omega))}{\partial W} = -\frac{1}{u'(w)}. \tag{B.5}$$

Moving these two expressions one period ahead, substituting them in (B.4), focusing on $\widetilde{p(\cdot)} > 0$ and $\pi_i > 0$ and rearranging we have that:

$$\frac{\partial \widetilde{p}(\Theta)}{\partial W_i} \frac{J(\Theta)}{\widetilde{p}(\Theta)} = \frac{1}{u'(w_i')} - \frac{1}{u'(w)},$$

with $\Theta \equiv (\phi(h, y), \tau + 1, W; \Omega')$ and where $w'$ is the wage next period in state $\Omega'$. $\square$

## B.5 Proof of Property 3.6

We discuss this property as a corollary of the following proposition.

**Proposition B.2.** *The Pareto frontier $J(h, \tau, \iota, W, y; a, \mu)$ is increasing in the aggregate productivity shock $a$, while retention probabilities, $\widetilde{p}(h, \tau, \iota, W; a, \mu)$ decrease in aggregate productivity.*

*Proof.* We proceed by backward induction.[34] Given the nature of the aggregate productivity process, the proposition is trivially true for workers $T$ periods old. Given that the firm increases its production while keeping the worker at least indifferent, $J$ is at least weakly increasing in $a$. However, the firm can also feasibly increase the worker's wage by $\varepsilon$, with $\varepsilon < \frac{\partial f(\cdot)}{\partial a}$. $J$ is thus strictly increasing in $y$.

Consider now a worker who is $T-1$ periods old. A firm matched to a worker in submarket $\{h, T-1, y, W_{y,\Omega}\}$ will face the following Pareto frontier,

$$J(h, T-1, y, W_y; a, \mu) = \sup_{w, W'_y} \Big( f(h, y; a) - w + \mathbb{E}_{\Omega} \left[ \widetilde{p}(h', T, W'_y; a', \mu')(f(h', y; a') - w') \right] \Big).$$

Let's assume that aggregate productivity increases from $\bar{a}$ to $\bar{a}+\varepsilon$ and that the firm keeps its policies constant once again. We aim at proving that, even in such a case, firm value increases while keeping the worker at least indifferent. If this is the case, it is *a fortiori* true that $J$ increases in $a$ after reoptimizing firms' policies. $\square$

**Corollary B.1.** *There exists a productivity threshold* $a^*(h, \tau, W_y, y)$ *below which firms will not continue the operate.*

*Proof.* The proof follows immediately from **Proposition B.2** and the timing of the shock. Given the timing of the shock, exit is fully determined by the current productivity shock and incumbent firms know in advance whether they are willing to produce in the next period.

Therefore, as the Pareto frontier is strictly increasing in $a$, firms are willing to continue the contract if $\mathbb{E}_{\Omega}[J(h', \tau+1, W'_{y,}, y; a', \mu')|h, \tau, W_y, y, a, \mu] \geq 0$, so that the threshold that determines exit is

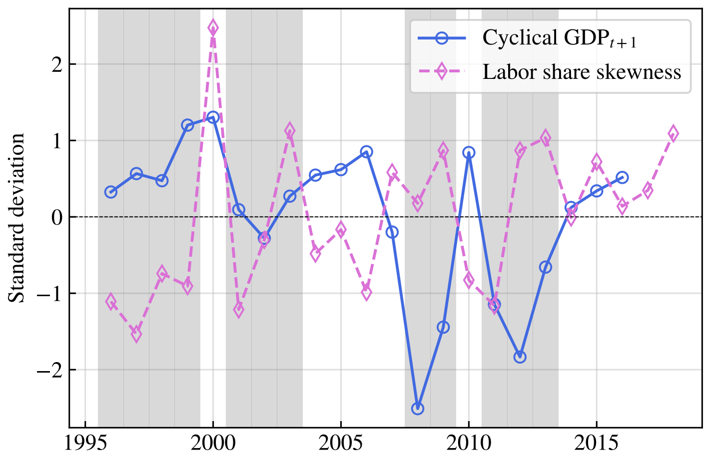
$$a^*(h, \tau, W_y, y) : \mathbb{E}_{\Omega}[J(h', \tau+1, W'_y, y; a', \mu')|h, \tau, W_y, y, a, \mu] = 0.$$

$\square$

[34] For compactness of notation, we omit without loss of generality the two-point lottery in the equations in the proof.

# C Additional Tables and Figures

**Figure C.1.** Skewness of Labor Share and GDP




**Note:** The figure plots the cyclical component of real GDP (Hamilton filter) and the skewness of labor share, both standardized. In the data, we compute the labor share as the ratio of labor costs to value added. Shaded areas are OECD based recessions. *Data source*: ISTAT and INPS-Uniemens.

**Table C.1.** E-U-E transitions

**(a)** Data

| Dep.Variable: Log-wage after E-U-E transition | (1) | (2) |
|---|---|---|
| Quality of origin firm (FQ): | | |
| $2^{nd}$ quint. | 0.100 | 0.128 |
| | (0.002) | (0.004) |
| $3^{rd}$ quint. | 0.143 | 0.210 |
| | (0.002) | (0.004) |
| $4^{th}$ quint. | 0.153 | 0.182 |
| | (0.002) | (0.004) |
| $5^{th}$ quint. | 0.230 | 0.260 |
| | (0.002) | (0.004) |
| Log-wage at origin | 0.669 | 0.609 |
| | (0.001) | (0.002) |
| Experience controls | ✓ | ✓ |
| Sex FE | ✓ | ✓ |
| Year FE | ✓ | ✓ |
| Contract type FE | ✓ | ✓ |
| Full- & Part-Time FE | ✓ | ✓ |
| Justified dismissals | ✓ | |
| $R^2$ | 0.48 | 0.36 |
| N | 955,602 | 338,975 |

**(b)** Model

| Dep. Variable: Log-wage after E-U-E transition | (1) |
|---|---|
| Quality of origin firm (FQ): | |
| $p0 \leq FQ < 75p$ | 0.743 |
| | (0.007) |
| $p75 \leq FQ < 100p$ | 0.783 |
| | (0.008) |
| Log-wage at origin | 0.117 |
| | (0.009) |
| Controls | ✓ |
| $R^2$ | 0.069 |
| N | 18,669 |

**Note:** Standard errors in parentheses. The tables report a specification on datasets based on workers that experience experience an Employment to Unemployment to Employment transitions (E-U-E). In Panel **(a)**, column (2) excludes separations that are justified in the Italian labor law (*giusta causa*). In the model, *Controls* include the pre-transition human capital and a polynomial in labor market experience. Referenced on page(s) [21] .